Lightning Protection of Buried Toll Cable

By E. D. SUNDE

A theoretical study of lightning voltages in buried telephone cable, of the liability of such cable to damage by lightning and of remedial measures, together with the results of simulative surge tests, oscillographic observations of lightning voltages and lightning trouble experience.

Introduction

PRACTICALLY all of the toll cable installed since 1939 has been of the carrier type and most of it has been buried in order to secure greater immunity from mechanical damage. It was realized, however, that burying the cable would not prevent damage due to lightning and that, on account of their smaller size, more damage was to be expected on the new carrier cables than on the much larger voice-frequency underground cables then in use. Moreover, when damage by lightning does occur, such as fusing of cable pairs or holes in the sheath, it is not so easy to locate and repair as on aerial cables, since excavations may have to be made at a number of points. Studies were therefore made of the factors affecting damage of buried cables by lightning and remedial measures were devised and put into effect in cases where a high rate of lightning failures was anticipated on new installations, or was experienced with cable already installed. Most of the cable installed was thus provided with extra core insulation, and shield wires were plowed in on many of the new routes.

It was recognized early in these studies that more effective lightning protection might be secured by providing the lead sheath with a thermoplastic coating of adequate dielectric strength and an outside copper shield, and that such cable might be required in territory where the earth resistivity is very high. This type of cable has recently been installed on a route in high-resistivity territory where experience has indicated that other types of construction would probably be inadequate and, since it has advantages also from the standpoint of corrosion and mechanical protection, it may be used also where lightning is not of such decisive importance.

When lightning strikes, the current spreads in all directions from the point where it enters the ground. If there are cables in the vicinity they will provide low resistance paths, so that much of the current will flow to the cables near the lightning stroke and in both directions along the sheath to remote points. The flow of current in the ground between the lightning channel and the cables may give rise to such a large voltage drop that the breakdown voltage of the soil is exceeded, particularly when the earth

resistivity is high. The lightning stroke will then arc directly to the cables from the point where it enters the ground, often at the base of a tree. Furrows longer than 100 feet have been found in the ground along the path of such arcs.

The current entering the sheath near the stroke point is attenuated as it flows towards remote points. Since a high earth resistivity is accompanied by a small leakage conductance between sheath and ground, the current will travel farther the larger the earth resistivity. The current along the sheath produces a voltage between the sheath and the core conductors, which is largest at the stroke point. This voltage is equal to the resistance drop in the sheath, between the stroke point and a point which is sufficiently remote so that the current in the sheath is negligible. Since the current travels farther along the sheath the higher the earth resistivity, this resistance drop will also increase with the earth resistivity. The maximum voltage between sheath and core is thus proportional to the sheath resistance and, as it turns out, to the square root of the earth resistivity. Carrier cables now being used are of smaller size and have a higher sheath resistance than full-size voice-frequency cables, and for this reason they are liable to have more lightning damage, particularly when the earth resistivity is high.

To secure experimental verification of certain points of the theory presented here, staged surge tests were made on the Stevens Point-Minneapolis cable, one of the first small-size buried toll cables to be installed. The results of these tests, which have already been published,[1] are here compared with those obtained theoretically, on the basis of the earth resistivity measured at the test location. Lightning voltages on this cable route were also recorded by automatic oscillographs and the results of these observations are also briefly discussed together with the rate of lightning failures experienced on this and other routes.

The first part of the paper deals with voltages between the cable conductors and the sheath due to sinusoidal currents and surge currents. The second part deals with the liability of damage due to excessive lightning voltages and with certain characteristics of lightning discharges of importance in connection with the present problem, such as the impedance encountered by the lightning channel in the ground, the rate of lightning strokes to ground and to buried structures and the crest current distribution for such strokes. In the third part remedial measures are discussed, together with lightning-resistant cable.

I. Voltages Between Cable Conductors and Sheath

1.1 *General*

Cable installed in the ground is designated "underground" when placed in duct, and "buried" when not in duct. Buried cable is sometimes pro-

vided with steel tape armor for protection against mechanical damage. While such armor may also reduce voltages due to low-frequency induction, mainly because of the high permeability of the steel, this is not true in the case of lightning voltages. The magnetic field in the armor due to lightning current in the cable is rather high, and the corresponding permeability fairly low. The armor resistance is, furthermore, quite high compared to that of the sheath, so that the effect of the armor may be neglected in considering lightning voltages. The tape or armor is usually separated from the sheath by paper and asphalt, but is bonded to the sheath at every splice point. Strokes to ground, or to the cable, may give rise to large currents in the armor and thus to excessive voltages between the armor and the sheath some distance from bonding points. The resulting arc may fuse a hole in the sheath or dent it, due to the explosive effect of the confined arc, and insulation failures may be experienced on this account. Such failures are not considered here since they are usually confined to a single point and are thus of less importance than insulation failures due to excessive voltages between the core conductors and the sheath, which may be spread for a considerable distance along the cable.

For protection against corrosion, buried cables are usually jute-covered (asphalt, paper and jute) and in some cases have thermoplastic or rubber coating. The leakance of jute-covered sheaths is usually large enough so that the cable may be assumed to be in direct contact with the earth and the leakance is, furthermore, increased at the time of lightning strokes by numerous punctures due to excessive voltage between sheath and ground. This effect is large enough so that even rubber-covered cable may be regarded as in direct contact with the soil in the case of direct strokes and sometimes also for strokes to ground in the vicinity of the cable, as discussed later.

In order to calculate the voltage between the sheath and the core conductors of a buried cable, due to a surge current entering the sheath or the ground in the vicinity of the cable, it is convenient to consider at first a sinusoidal current. The voltage due to a unit step current may then be obtained by operational solution and, in turn the voltage for a current $J(t)$ of arbitrary wave shape, by means of either one of the integrals:

$$\begin{aligned} V(t) &= \int_0^t J'(t - \tau)S(\tau)\, d\tau \\ &= \int_0^t J(t - \tau)S'(\tau)\, d\tau \end{aligned} \tag{1}$$

where $S(t)$ is the voltage due to unit step current and $S'(t)$ the time derivative of this voltage. The second of the above integrals is more convenient in the present case.

Photographic observations[2] indicate that a lightning discharge is usually initiated in the cloud by a so-called "stepped leader," except in the case of discharges to sufficiently tall structures[3] where this leader is initiated at the ground end. After the leader reaches the ground, or the cloud in the case of a tall structure, a heavy current "return stroke" proceeds from the ground toward the cloud at about $\frac{1}{10}$ the velocity of light. The main surge of current in the return stroke, which usually lasts for less than 100 microseconds, may be followed by a low current lasting for $\frac{1}{10}$ second or so.

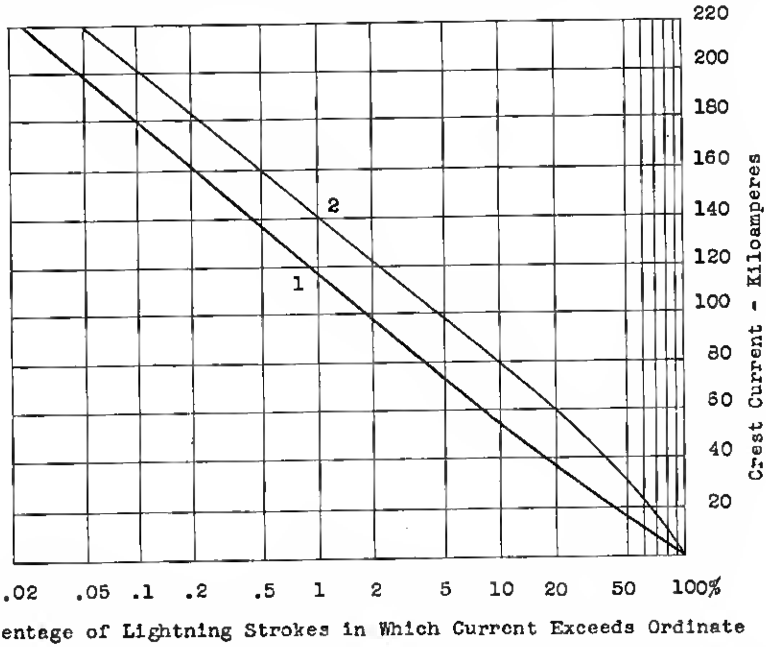

Figure 1—Distribution of crest currents in lightning strokes.

Curve 1: Currents in strokes to transmission line ground structures, based on 4410 measurements, 2721 in U. S. and 1689 in Europe.

Curve 2: Currents in strokes to buried structures, derived from curve 1.

There may then be a second leader, which does not exhibit the stepped character of the first leader and always proceeds from the cloud, and a second return stroke. This may be followed by a third leader and so on, the average number of strokes in multiple discharges being about 4 and the average time interval between strokes about $\frac{1}{10}$ second. Single-stroke discharges are, however, most common, discharges having more than 6 strokes being quite rare although discharges with as many as 40 strokes have been observed.

The crest value of currents in lightning discharges varies over wide limits.

Measurements of current in the ground structure of transmission lines[4,5] indicate that a relationship as shown in Fig. 1 exists between the crest currents and the percentage of discharges in which they occur. In the same figure is shown the crest current distribution for strokes to buried structures, which is derived in Part II from the curve for strokes to transmission line ground structures. Although measurements of wave shape are not extensive, they indicate that the current reaches its crest value in 5 to 10 microseconds and that it decays to half its maximum in 25 to 100 microseconds, the average being about 50 microseconds.[6] An average wave shape is assumed in this investigation.

In some 80 per cent of all lightning discharges the cloud is negative, so that the flow of current is from the earth toward the cloud. Further details about lightning discharges are summarized in recent surveys[6,7] which also contain an extensive list of references.

1.2 *Direct Strokes—Current Propagation Along Sheath*

As mentioned in the introduction and discussed further in Part II, a lightning stroke to ground may arc to a buried cable in the vicinity, in which case virtually all of the current will enter the sheath near the stroke point.

When a sinusoidal current J enters the sheath at $x = 0$ and the sheath is assumed to extend indefinitely in opposite directions from this point, the sheath current at the distance x is given by the following approximate expression

$$I(x) = \frac{J}{2} e^{-\Gamma x} \tag{2}$$

where Γ is the propagation constant of the sheath-earth circuit and is given by the following expression, derived in a previous paper.[8]

$$\Gamma = \frac{1}{v} [i\omega(i\omega + 1/\rho\kappa)]^{\frac{1}{2}} \tag{3}$$

where: v = Velocity of propagation along sheath
$= (2/\nu\kappa)^{\frac{1}{2}}$ meters per second
ν = Inductivity of earth $= 1.256 \cdot 10^{-6}$ hy/meter
κ = Capacitivity of earth $= \epsilon \cdot 8.858 \cdot 10^{-12}$ fd/meter
ϵ = Dielectric constant of earth
ρ = Earth resistivity, meter-ohms

In deriving the above formula the resistance of the sheath is neglected in comparison with its external reactance, which is permissible for frequencies in the range of importance, and the sheath is assumed to be half buried, that is, with its axis in the plane of the earth's surface. The latter assumption gives rise to a comparatively small error when the formula is applied

to cables buried at depths up to one meter or so, the propagation constant for a cable buried at infinite depth being larger than that given above by a factor of $\sqrt{2}$.

It is assumed that the current is propagated from the cable up the lightning channel with infinite velocity. The voltage between the cable conductors and the sheath obtained in this manner reaches a crest value after some 50 to 100 microseconds, or after the current in an actual lightning channel has traveled from the ground to the cloud. The error due to this assumption is thus probably quite small as regards the crest voltage, although the wave front will be somewhat slower when the actual velocity of propagation is considered.

1.3 *Direct Strokes—Voltage for Sinusoidal Current*

The current along the sheath gives rise to an electric force along the latter. The electric force along the inner surface of the sheath is given by:

$$E(x) = zI(x) = \frac{J}{2} ze^{-\Gamma x} \tag{4}$$

where z is the mutual-impedance of the sheath-earth and core-sheath circuits.

The latter mutual impedance is equal to the ratio of electric force along the inner surface of the sheath at any point, to the total current along the sheath at the same point, and for low frequencies equals the direct-current resistance of the sheath. It is given by the following slightly approximate formula:[9]

$$Z = R(i\omega\gamma)^{\frac{1}{2}}/\sinh\,(i\omega\gamma)^{\frac{1}{2}} \tag{5}$$

where: $\omega = 2\pi f$ and
R = Unit length d-c resistance of sheath, ohms/meter
$\gamma = \nu\delta/2\pi a R$
δ = Thickness of sheath, meter
a = Radius of sheath, meter
ν = Intrinsic Inductivity of sheath
$= 1.256 \times 10^{-6}$ henrys/meter

The current in the core-sheath circuit and the voltage between core and sheath due to an impressed field $E(x)$ along the core (inner surface of sheath) are obtained from the following equations, which are the general solutions of the transmission line equation for the core-sheath circuit.

$$J(x) = [A + P(x)]e^{-\Gamma_0 x} - [B + Q(x)]e^{\Gamma_0 x} \tag{6}$$

$$U(x) = K_0[A + P(x)]e^{-\Gamma_0 x} + K_0[B + Q(x)]e^{\Gamma_0 x} \tag{7}$$

where Γ_0 and K_0 are the propagation constant and the characteristic impedance of the core-sheath circuit and

$$P(x) = \frac{1}{2K_0}\int_0^x E(x)e^{\Gamma_0 x}\,dx = \frac{z}{2K_0}\,\frac{1 - e^{-(\Gamma-\Gamma_0)x}}{\Gamma - \Gamma_0} \tag{8}$$

$$Q(x) = \frac{1}{2K_0}\int_0^x E(x)e^{-\Gamma_0 x}\,dx = \frac{z}{2K_0}\,\frac{1 - e^{-(\Gamma+\Gamma_0)x}}{\Gamma - \Gamma_0} \tag{9}$$

Since the current must be zero at $x = 0$, it is necessary that $A = B$. To make the curent vanish when x becomes infinity, B must equal $-Q(\infty) = -\frac{z}{2K_0}\,\frac{1}{\Gamma + \Gamma_0}$. Witb these boundary conditions the voltage between core and sheath becomes:

$$U(x) = \frac{J}{2}\,\frac{z}{\Gamma^2 - \Gamma_0^2}\,(\Gamma e^{-\Gamma x} - \Gamma_0 e^{-\Gamma_0 x}) \tag{10}$$

The propagation constant Γ_0 is much smaller than Γ, and may be taken as:

$$\begin{aligned}\Gamma_0 &= [(R_0 + i\omega L_0)i\omega C_0]^{\frac{1}{2}} \\ &= \frac{1}{v_0}\,[i\omega(i\omega + R_0/L_0)]^{\frac{1}{2}}\end{aligned} \tag{11}$$

R_0 = Unit length resistance of core-sheath circuit, ohms/meter
L_0 = Unit length inductance of core-sheath circuit, hy/meter
C = Unit length capacitance of core-sheath circuit, fd/meter
$v_0 = (1/L_0C_0)^{\frac{1}{2}}$

1.4 *Direct Strokes—Lightning Voltage at Stroke Point*

The largest voltage between sheath and core conductors is obtained for $x = 0$, and for this case (10) becomes:

$$\begin{aligned}U(0) &= \frac{J}{2}\,\frac{z}{\Gamma + \Gamma_0} \\ &= \frac{J}{2}\,\frac{R\gamma^{\frac{1}{2}}/\sinh\,(i\omega\gamma)^{\frac{1}{2}}}{\frac{1}{v}\,(i\omega + 1/\rho\kappa)^{\frac{1}{2}} + \frac{1}{v_0}\,(i\omega + R_0/L_0)^{\frac{1}{2}}}\end{aligned} \tag{12}$$

For sufficiently high frequencies, so that $(i\omega\gamma)^{\frac{1}{2}} \gg 1$, $i\omega > 1/\rho\kappa$, $i\omega > R_0L_0$, and $\sinh\,(i\omega\gamma)^{\frac{1}{2}} \leqq \frac{1}{2}\exp\,(i\omega\gamma)^{\frac{1}{2}}$ expression (12) becomes

$$U(0) = JR\gamma^{\frac{1}{2}}\,\frac{vv_0}{v + v_0}\,(i\omega)^{-\frac{1}{2}}e^{-(i\omega\gamma)^{\frac{1}{2}}} \tag{13}$$

For sufficiently low frequencies, so that $(i\omega\gamma)^{\frac{1}{2}} < 1$, $i\omega < i/\rho\kappa$, $i\omega < R_0/L_0$, and $\sinh (i\omega\gamma)^{\frac{1}{2}} \cong (i\omega\gamma)^{\frac{1}{2}}(1 + i\omega\gamma/6)$, expression (12) becomes

$$U(0) = \frac{J}{2} \frac{R}{\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}} \frac{1}{(i\omega)^{\frac{1}{2}}(i + \omega\gamma/6)^{\frac{1}{2}}} \tag{14}$$

where

$$\alpha = \nu/2\rho, \qquad \beta = R_0C_0$$

For small values of time, corresponding to large values of $i\omega$, the function $S'(t)$ defined before, as obtained by operational solution of (13) is [10]

$$S'(t) = R\gamma^{\frac{1}{2}} \frac{vv_0}{v + v_0} \left(\frac{1}{\pi t}\right)^{\frac{1}{2}} e^{-\gamma/4t} \tag{15}$$

For large values of time, corresponding to small values of $i\omega$, the function is obtained by operational solution of (14) and equals: (Reference 10, pair 542)

$$S'(t) = \frac{R}{2(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}})} \left(\frac{6}{\gamma}\right)^{\frac{1}{2}} h(\sqrt{6t/\gamma}) \tag{16}$$

where, with $(6t/\gamma)^{\frac{1}{2}} = u$:

$$h(u) = -ie^{-u^2} \operatorname{erf}(iu) = \frac{2}{\sqrt{\pi}} e^{-u^2} \int_0^u e^{\tau^2} d\tau \tag{17}$$

erf being the error function.

Values of the function $h(u)$ are given in Table I.

In Fig. 2, curve 1 shows the function $S'(t)$ calculated from (15), and curve 2 that calculated from (16), for a cable of 1.4″ diameter, using constants as indicated in figure. The constants apply to a cable on which measurements have been made of the voltage between sheath and core conductors, at a location where the measured earth resistivity was 400 meter-ohms. The function $S'(t)$ corresponding to equation (12) is obtained with sufficient accuracy by drawing a transition curve, 3, between curves 1 and 2.

If the impedance z is taken equal to the direct-current resistance R of the sheath and if the velocity of propagation along the sheath and along the core are assumed to be infinite, so that $\Gamma = (i\omega\alpha)^{\frac{1}{2}}$ and $\Gamma_0 = (i\omega\beta)^{\frac{1}{2}}$, the following expression is obtained

$$S'(t) = \frac{R}{2(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}})} \left(\frac{1}{\pi t}\right)^{\frac{1}{2}} \tag{18}$$

In the following it will be shown that (18) is accurate enough for practical purposes.

The wave shape of the current in lightning strokes may be approximated by an expression of the form:

$$J(t) = I(e^{-at} - e^{-bt}). \tag{19}$$

With $a = .013 \cdot 10^6$, $b = .5 \cdot 10^6$, a current of the wave shape used in the measurements referred to above is obtained. This current reaches its crest in 10 microseconds and decays to its half-value in 65 microseconds, and is fairly representative of the average wave shape of lightning stroke currents. In the following, the voltages are for convenience referred to a crest value of 1000 amperes, which is obtained when $I = 1150$ amperes, the latter current being the initial value of each of the two exponential component currents included in (19).

Table I

$$\sqrt{\frac{\pi}{2}}\, h(u) = e^{-u^2} \int_0^u e^{\tau^2}\, d\tau = -\sqrt{\frac{\pi}{2}}\, ie^{-u^2} \operatorname{erf}(iu)$$

$$\cong u \text{ when } u < .1$$

$$\cong \frac{1}{2u} \text{ when } u > 10$$

u	$\sqrt{\frac{\pi}{2}}\, h(u)$	u	$\sqrt{\frac{\pi}{2}} \cdot h(u)$
0	0	1.5	.4283
.1	.0993	2.	.3014
.2	.1948	2.5	.2232
.3	.2826	3.0	.1782
.4	.3599	3.5	.1496
.5	.4244	4.	.1293
.6	.4748	4.5	.1141
.7	.5105	5.	.1021
.8	.5321	6.	.0845
.9	.5407	8.	.0630
1.0	.5381	10.	.0503

A more complete table for the range between $u = 0$ and $u = 4$ is published in *Berichteverhandlungen Akademie der Wissenschaften, Leipzig, Math-Phys. Klasse*, Vol. 80, 1928, pages 217 to 223.

In Fig. 3, the dashed curve shows the measured voltage and curves 1 and 2 that calculated for the above surge current for two conditions. In calculating curve 1, $S'(t)$ was taken as curve 3 of Fig. 2, the voltage being obtained by numerical integration in accordance with (1); in calculating curve 2, z is taken as the direct-current resistance of the sheath and the velocities of propagation are assumed to be infinite, so that $S'(t)$ is given by (18). In the latter case the following expression is obtained for the voltage by solution of (1):

$$V(t) = \frac{JR}{2(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}})} [a^{-\frac{1}{2}} h(\sqrt{at}) - b^{-\frac{1}{2}} h(\sqrt{bt})] \tag{20}$$

where the function $h(u)$, $u = \sqrt{at}$ or $\sqrt{bt}$, is defined as before.

Comparison of curves 1 and 2 of Fig. 3 shows that (20) is accurate enough for practical purposes, so that the voltage may be taken proportional to the direct-current resistance of the sheath. Since $\beta^{\frac{1}{2}}$ is only 2.5 per cent of $\alpha^{\frac{1}{2}}$, propagation in the core-sheath circuit may be neglected in comparison with propagation along the sheath-earth circuit, so that it is permissible to take the voltage proportional to the square root of the earth resistivity.

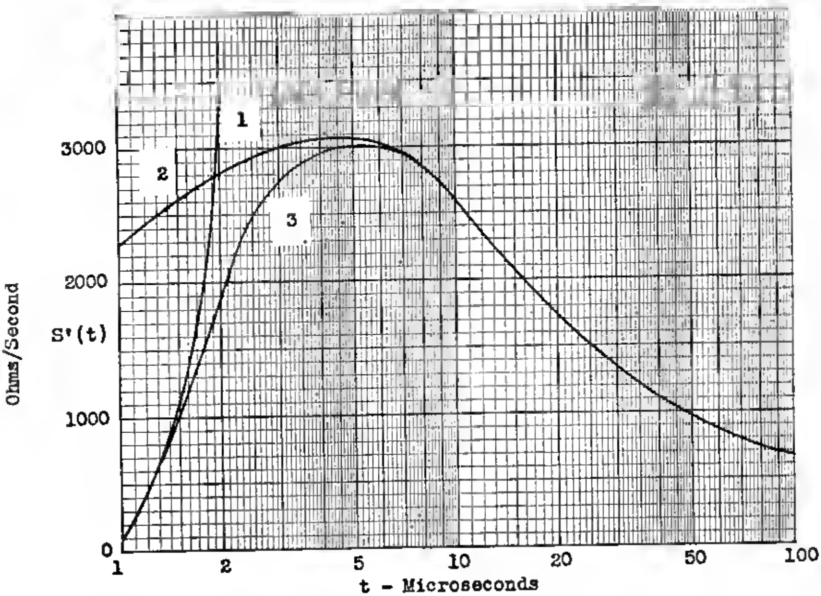

Figure 2—Approximate solution for $S'(t)$.

1: Calculated from formula for small times.
2: Calculated from formula for large times.
3: Transition curve giving approximate solution for $S'(t)$.
Earth resistivity, ρ = 400 meter-ohms.
Radius of cable, a = 1.75 cm.
Sheath thickness, δ = 2.4 mm.
Sheath resistance, $R = .92 \cdot 10^{-3}$ ohms/meter.
Core-sheath cap. $C_0 = .96 \cdot 10^{-9}$ fd/meter.
Core-sheath resist. $R_0 = R \cong .92 \cdot 10^{-3}$ ohm/meter.
Velocity $v_0 = 2 \cdot 10^8$ meter/sec.
Velocity $v = 1 \cdot 10^8$ meter/sec.

Furthermore, from (20) it is seen that when a and b are divided by the same factor k, so that the wave shape of the current remains the same but the duration of the current is increased k times, the voltage is increased $\sqrt{k}$ times. Thus, if the surge current had reached its crest value in 20 microseconds and its half-value in 130 microseconds, the voltage would be increased by $\sqrt{2}$, and the crest voltage would have been reached after 120 rather than 60 microseconds.

If the breakdown voltage of the core insulation is assumed to be 2000 volts, the above cable would be able to withstand a stroke current of about 30,000 amperes before the insulation is punctured. From Fig. 1 it is seen

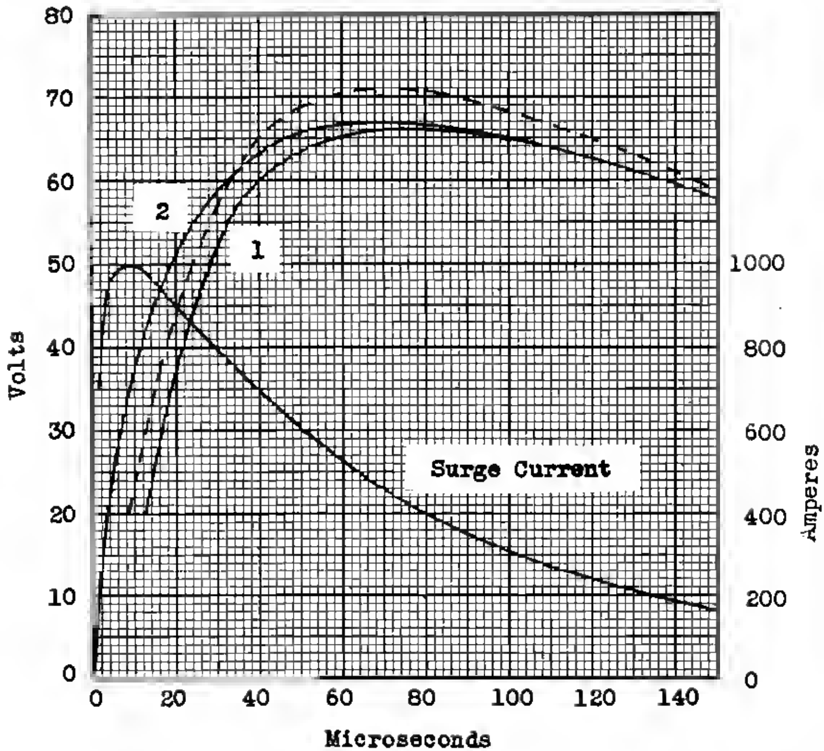

Figure 3—Comparison of measured, shown by dashed curve, and calculated voltage between sheath and core conductors, shown by curves 1 and 2, for surge current as shown and cable constants as given in Fig. 2.

1: Calculated from formula including skin-effect in sheath and finite velocity of propagation.

2: Calculated from formula based on d-c resistance of sheath and assuming infinite velocity of propagation.

that in about 50 per cent of all strokes the crest current exceeds 30,000 amperes.

When there are two cables, each will provide shielding for the other, and the shielding effect may be calculated as for shield wires (Sec. 3.3). Frequently the cables are of equal or nearly equal size and are close together. It is then accurate enough to use the parallel resistance of the two sheaths in calculating the voltage, which will be practically the same in both cables.

1.5 *Direct Strokes—Lightning Voltages Along Cable*

It was shown above that with only a minor error the impedance z may be taken equal to the direct-current resistance of the sheath and that the propagation constants may be taken as

$$\Gamma = (i\omega\alpha)^{\frac{1}{2}}, \qquad \Gamma_0 = (i\omega\beta)^{\frac{1}{2}}$$

With this modification expression (10) becomes:

$$U(x) = \frac{J}{2}\frac{R}{\alpha - \beta}[\alpha^{\frac{1}{2}} e^{-(i\omega\alpha)^{\frac{1}{2}}x} - \beta^{\frac{1}{2}} e^{-(i\omega\beta)^{\frac{1}{2}}x}](i\omega)^{-\frac{1}{2}} \tag{21}$$

The corresponding function S' is:[10]

$$S'(x, t) = \frac{R}{2(\alpha - \beta)}\left(\frac{1}{\pi t}\right)^{\frac{1}{2}}(\alpha^{\frac{1}{2}} e^{-\alpha x^2/4t} - \beta^{\frac{1}{2}} e^{-\beta x^2/4t}) \tag{22}$$

The voltage due to a surge current $J(t)$, as obtained from (1), may be expressed as:

$$V(x, t) = \frac{R}{2(\alpha - \beta)}[\alpha^{\frac{1}{2}} g(\alpha^{\frac{1}{2}} x, t) - \beta^{\frac{1}{2}} g(\beta^{\frac{1}{2}} x, t)] \tag{23}$$

where, with $\sigma = \alpha^{\frac{1}{2}}x$ or $\beta^{\frac{1}{2}}x$,

$$g(\sigma, t) = \int_0^t J(t - \tau)\left(\frac{1}{\pi\tau}\right)^{\frac{1}{2}} e^{-\sigma^2/4\tau}\, d\tau \tag{24}$$

For a current as given by (19), the latter integral may be expressed in terms of error functions of complex arguments, for which, however, no tables are available at present. Curves for the function g, as obtained by numerical integration are shown in Fig. 4.

When the core conductors are connected to the sheath at $x = 0$, the constants A and B of (6) and (7) are obtained from the following boundary conditions: At $x = 0$, $V(0) = 0$ so that $A = -B$. As before, $B = -Q(\infty)$. The voltage between core and sheath is then given by:

$$\begin{aligned} U_0(x) &= \frac{J}{2}\frac{R\Gamma}{\Gamma^2 - \Gamma_0^2}(e^{-\Gamma x} - e^{-\Gamma_0 x}) \\ &= \frac{JR\alpha^{\frac{1}{2}}}{\Gamma^2 - \Gamma_0^2}[e^{-(i\omega\alpha)^{\frac{1}{2}}x} - e^{-(i\omega\beta)^{\frac{1}{2}}x}] \end{aligned} \tag{25}$$

In this case the derivative of the voltage due to unit step current is:

$$S_0'(x, t) = \frac{R\alpha^{\frac{1}{2}}}{2(\alpha - \beta)}\left(\frac{1}{\pi t}\right)^{\frac{1}{2}}[e^{-\alpha x^2/4t} - e^{-\beta x^2/4t}] \tag{26}$$

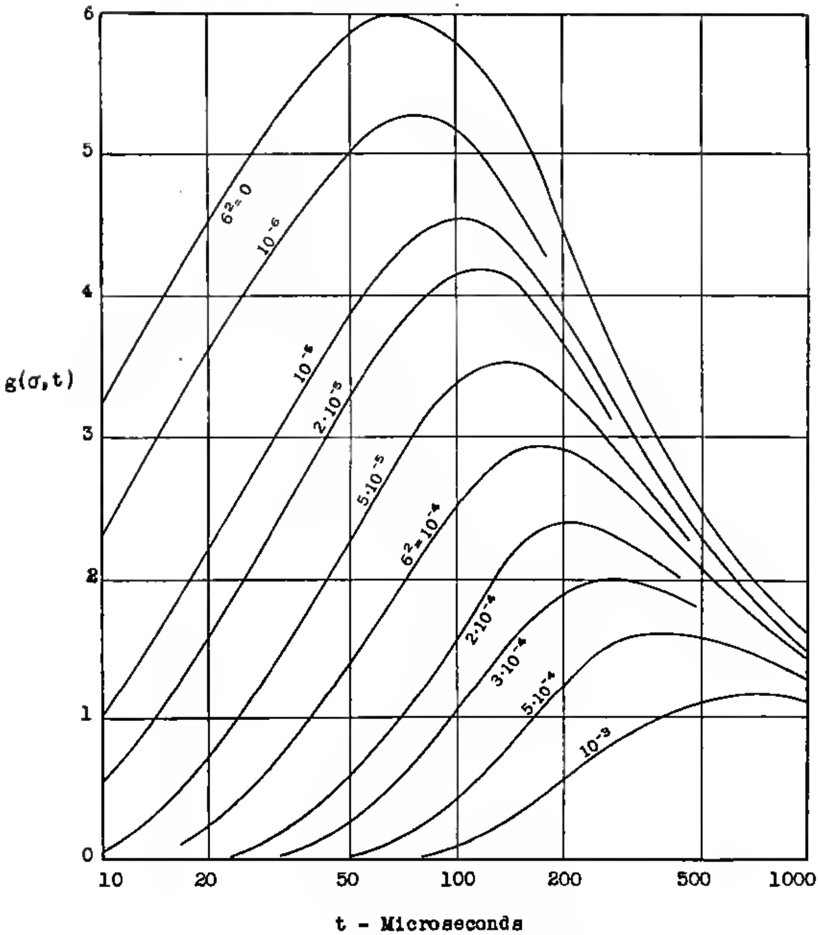

Figure 4—Function $g(\sigma, t) = \int_0^t J(t - \tau) \left(\frac{1}{\pi\tau}\right)^{\frac{1}{2}} e^{-\sigma^2/4\tau} \, d\tau$

$J(t) = I(e^{-at} - e^{-bt})$

$a = 1.3 \cdot 10^4$, $b = 5 \cdot 10^5$ $\quad I = 1150$ amperes

and

$$V_0(x, t) = \frac{R\alpha^{\frac{1}{2}}}{2(\alpha - \beta)} [g(\alpha^{\frac{1}{2}} x, t) - g(\beta^{\frac{1}{2}} x, t)] \tag{27}$$

In Fig. 5 the crest values of $V(x, t)$ and $V_0(x, t)$, calculated for the cable considered before, are plotted against x, together with those observed in the tests. When the voltage $V(0, t)$ at the point where current enters the sheath is great enough to break down the insulation of a core conductor, the

latter will be in contact with the sheath by virtue of arcing. Under this condition, the voltage $V_0(x, t)$ between this conductor and the sheath will increase with distance along the cable as shown in Fig. 5. A maximum is reached a fairly short distance from the original fault, and beyond this point the voltage slowly decreases. After a puncture of the insulation where current enters the sheath, other failures may therefore occur, not necessarily at the point where $V_0(x, t)$ is largest, but sometimes at points nearer or much farther away where the insulation may be weaker. A single lightning stroke may thus cause insulation failures over a considerable distance along the cable.

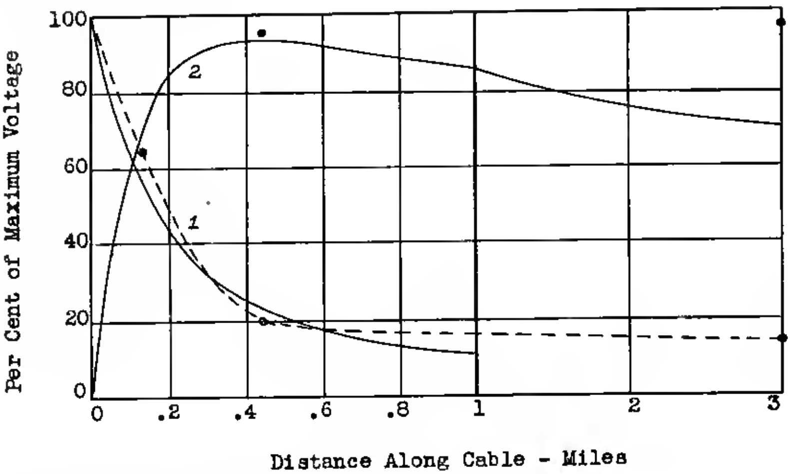

Figure 5—Comparison of measured variation of voltage between sheath and core, along cable; as shown by points and dashed curve, with calculated variation shown by curves 1 and 2.
1: Conductor not connected to sheath.
2: Conductor connected to sheath at point where surge current enters sheath.

1.6 *Direct Strokes—Voltage Due to Long Duration Current*

As mentioned before, a current of low value and long duration may exist on the lightning channel after the main discharge. This current is usually of such long duration that the resistance of the sheath must be considered in calculating the current propagation along the cable. The propagation constant in that case becomes

$$\Gamma = [(R + i\omega L)G]^{\frac{1}{2}} = \left(\frac{R}{L} + p\right)^{\frac{1}{2}} \left(\frac{\nu}{2\rho}\right)^{\frac{1}{2}} \tag{28}$$

R, L and G being the unit length resistance, inductance and leakance of the sheath-earth circuit. Neglecting propagation in the core-sheath circuit,

the voltage between core and sheath at the stroke point due to a sinusoidal current J_1 becomes,

$$U_1(0) = \frac{J_1 R}{2\Gamma} = J_1\left(\frac{\rho}{2\nu}\right)^{\frac{1}{2}} R(p + R/L)^{-\frac{1}{2}} \tag{29}$$

The corresponding voltage for a unit step current J_1 is:

$$V_1(0, t) = J_1 R\left(\frac{\rho L}{2\nu R}\right)^{\frac{1}{2}} \text{erf } (Rt/L)^{\frac{1}{2}} \tag{30}$$

where erf is the error function.

For large values of time, when $Rt/L > 1$, (30) becomes

$$V_1(0, t) = J_1 R\left(\frac{\rho L}{2\nu R}\right)^{\frac{1}{2}} \tag{31}$$

The latter expression is valid when t exceeds about 2 milliseconds and thus applies for the long duration current of a lightning stroke, since the latter usually lasts for about 100 milliseconds. For a current of 1000 amperes, the core-sheath voltage for a cable of 1.4″ diameter is about 700 volts. In many strokes the long duration current may be several hundred amperes, and a substantial voltage may then exist between core and sheath for .1 second or so. Thus, while this current component does not increase the crest voltage, it substantially increases the likelihood of permanent failure when the insulation is punctured by prolonging the current through the puncture.

1.7 *Strokes to Ground Not Arcing to Cable*

Let it be assumed that the current enters the ground at the distance y from a buried cable and that conditions are such that it does not arc to the latter. The flow of current in the ground gives rise to an electric force in the ground along the cable, and thus to currents in the sheath and to voltages between core and sheath. When the earth is assumed to have uniform conductivity, the earth potential at the distance r from the point where current enters the ground is given by:

$$Ve = JQ_0(r) = J\rho/2\pi r \tag{32}$$

where

ρ = Earth resistivity in meter-ohms
$r = (x^2 + y^2)^{\frac{1}{2}}$ = Distance in meters

The sheath current and the voltage between sheath and core may in this case be obtained from published formulas,[11] provided propagation along

the core-sheath circuit is neglected in comparison with propagation along the sheath-earth circuit, which is permissible. The voltage between sheath and core conductor differs by the factor z/Z from the voltage between sheath and ground as given in Table II, case 3 of the paper referred to, z being defined as before and Z being the unit length self-impedance of the sheath-earth circuit. At a point opposite the lightning stroke, $x = 0$, the voltage between core and sheath is in this case given by:

$$U(0, y) = J\frac{R\Gamma}{Z}\int_0^\infty Q_0(r)e^{-x}\,dx = J\frac{R}{Z}\frac{\rho}{2\pi}\Phi(\Gamma y) \tag{33}$$

where $Z = \Gamma^2/G$ and G is the unit length leakance of the sheath-earth circuit. The leakance is given by the approximate expression:

$$G = \left(\frac{\rho}{\pi}\log\frac{1}{\Gamma a}\right)^{-1} \tag{34}$$

a being the radius of the sheath and $\log = \log_\epsilon$.

The function $\Phi(\Gamma y)$ is given by the approximate formula:

$$\Phi(\Gamma y) \cong \log\frac{1+\Gamma y}{\Gamma y} \tag{35}$$

Inserting (34) and (35) in (33), the latter expression may be written:

$$U(0, y) = U(0, a)\lambda(\Gamma y) \tag{36}$$

where $U(0, a) = U(0)$ is the voltages when the current enters the sheath directly $(y = a)$ and:

$$\lambda(\Gamma y) = \left(\log\frac{1+\Gamma y}{\Gamma y}\right)\Big/\log 1/\Gamma a \tag{37}$$

where $\Gamma = (i\omega\nu/2\rho)^{\frac{1}{2}} = (i\omega\alpha)^{\frac{1}{2}}$.

The rigorous solution of the time function corresponding to (36) would be rather complicated. Since, however, λ is the ratio of two functions, each of which varies logarithmically with Γ, and thus varies only slightly with $i\omega$, an approximate solution is obtained by replacing $i\omega$ with $1/t$ in (37). For instance, the solution of an operational expression p^{-n} is $t^n/n!$ while the solution of $p^{-n}\log p$ is $[\psi(1+n) + \log 1/t]t^n/n!$, ψ being the logarithmic derivative of the gamma function. For representative values of n and $t(n < 1, t < 10^{-4})$, ψ is less than 5% of $\log 1/t$, so that a good approximation is obtained by replacing p by $1/t$ in $\log p$, which in this illustration simulates the factor $\lambda(\Gamma y)$. With this approximation:

$$V(0, y, t) = V(0, a, t)\lambda[y(\alpha/t)^{\frac{1}{2}}] \tag{38}$$

In Fig. 6 is shown the variation in the voltage calculated from (38), together with that observed in the tests referred to before. That the measured decrease in the voltage is smaller than calculated is due to the fact that the earth resistivity at the test location increases with depth. Earth resistivity measurements made by the four-electrode method show that the resistivity is about 400 meter-ohms for electrode spacings up to about 20 feet and then gradually increases, reaching about 700 meter-ohms at 300 feet, 1200 meter-ohms at 1000 feet and approaching 1500 meter-ohms for

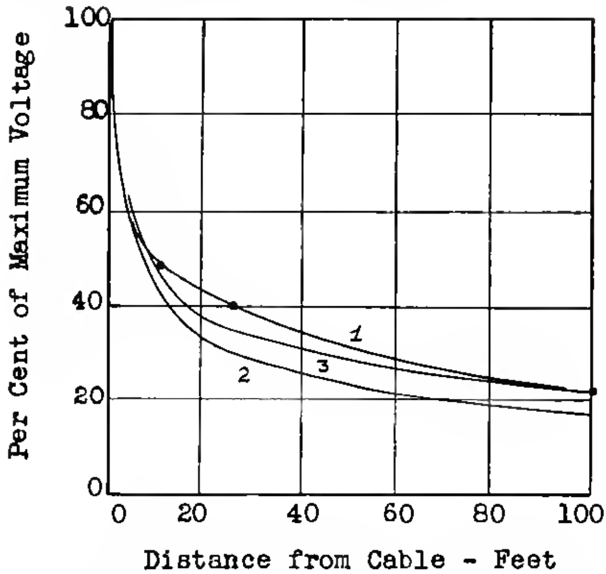

Figure 6—Reduction in voltage between sheath and core with increasing distance from cable to point where current enters the ground.
1: Measured when remote ground representing cloud is at a distance of 1000 ft.
2: Calculated for uniformly conducting earth with remote ground at distance of 1000 ft.
3: Calculated for uniformly conducting earth with remote ground at infinity.

large electrode spacings. The measured variation in voltage with separation is in substantial agreement with that calculated for an earth structure of this type in the manner outlined in Section 1.9.

1.8 *Discharges Between Clouds*

In considering voltages due to discharges between clouds, the lightning channel is assumed to parallel the cable. Due to magnetic induction, the lightning current will give rise to an impressed electric force along the cable sheath. Without much error it may be assumed that there is no impressed force outside the exposed section of the sheath and that the electric force in the exposed section due to a sinusoidal current J is $E^0(x) =$

JM, where M is the unit length mutual impedance of the lightning channel and the sheath. The resulting sheath current $I(x)$ is obtained from (6) and (7), when E is replaced by E^0, Γ_0 by Γ and K_0 by K, the characteristic impedance of the sheath-earth circuit. The constants A and B are found by observing the voltage between sheath and ground is zero at $x = s/2$. The electric force along the core is given by $E(x) = RI(x)$, and the voltage between the core conductors and the sheath is obtained by a second application of (6) and (7), the constants A and B being determined from the condition that the latter voltage must equal zero at $x = s/2$. The voltage between the core conductors and the sheath at the distance x along the cable beyond one end or the other of the lightning channel projection on the cable is then:

$$U(x) = \frac{JRM\Gamma^2 s}{2Z(\Gamma^2 - \Gamma_0^2)}\left[\frac{e^{-\Gamma_0 x}}{\Gamma_0 s}(1 - e^{-\Gamma_0 s}) - \frac{e^{-\Gamma x}}{\Gamma s}(1 - e^{-\Gamma s})\right] \quad (39)$$

the sign of the voltage beyond one end of the channel being opposite to that beyond the other end.

Since $\Gamma \gg \Gamma_0$, the last bracket term may be neglected. It was shown previously, that attenuation along the core-sheath circuit within a distance of one mile, which is representative of the length s, is quite small, so that $1 - e^{-\Gamma_0 s} \cong \Gamma_0 s$. With these modifications:

$$U(x) = \frac{JRM\Gamma^2 s}{2Z(\Gamma^2 - \Gamma_0^2)} e^{-\Gamma_0 x} \quad (40)$$

The earth-return impedances M and Z are given by the following approximate expressions:[12]

$$M = \frac{i\omega v}{2\pi} \frac{\sqrt{2}h}{(h^2 + y^2)(i\omega\alpha)^{\frac{1}{2}}} \quad (41)$$

$$Z = \frac{i\omega v}{2\pi} \log \frac{\sqrt{2}}{a(i\omega\alpha)^{\frac{1}{2}}} \quad (42)$$

where α is defined as before, $\log = \log_\epsilon$ and:

h = height of lightning channel above ground

y = horizontal separation of lightning channel from cable

The expression for M holds when $\alpha(h^2 + y^2)^{\frac{1}{2}} > 5$, a condition which is satisfied in the important part of the frequency range.

Inserting (41) and (42) in (40):

$$U(x) = \frac{JR\alpha^{\frac{1}{2}}}{2(\alpha - \beta)} \mu \left(\frac{1}{i\omega}\right)^{\frac{1}{2}} e^{-(i\omega\beta)^{\frac{1}{2}} x} \quad (43)$$

where

$$\mu = \frac{\sqrt{2}hs}{(h^2 + y^2) \log \frac{\sqrt{2}}{a(i\omega\alpha)^{\frac{1}{2}}}} \tag{44}$$

Comparison with (21) and (23) shows that in this case:

$$V(x, t) = \frac{R\alpha^{\frac{1}{2}}}{2(\alpha - \beta)} \mu g(\beta^{\frac{1}{2}} x, t) \tag{45}$$

In the above solution, μ was assumed constant. Actually it changes slightly with frequency and, for reasons mentioned before, it is accurate enough for practical purposes to replace $i\omega$ with $1/t$ when calculating μ.

The maximum voltage is obtained at $x = 0$, i.e., at a point opposite one end or the other of the lightning channel, and comparison with (22) shows that this voltage differs from that obtained in the case of a direct stroke by the factor μ, since $\beta^{\frac{1}{2}} \ll \alpha^{\frac{1}{2}}$ so that the second term may be neglected in (23). The above factor has the following approximate value:

$$\mu = .14 \frac{sh}{h^2 + y^2} \tag{46}$$

Since each cloud has an equal and oppositely charged image at the distance h below the surface of the ground, the electric field between cloud and ground is substantially equal to that between the clouds when $s = 2h$. For a discharge to take place between clouds, rather than to the earth, the length s would, therefore, have to be less than $2h$; so that with $y = 0$ the factor would not be expected to exceed $\mu = .28$. Thus, for the cable previously considered, failures due to discharges between clouds would not be expected except for currents in excess of 100,000 amperes in a lightning channel approximately above and parallel to the cable.

Maximum voltages of opposite signs are obtained at the two ends of the lightning channel, the voltage at the mid-point being zero. As the distance x from one end of the lightning channel increases, the voltage diminishes rather slowly in the same manner as shown in Fig. 5 for $V_0(x, t)$.

1.9 *Stratified Earth Structures*

In the foregoing, the earth was assumed to have a uniform resistivity ρ. In many cases the average resistivity near the surface along a route may be substantially greater or smaller than the resistivity at greater depths, and this condition may affect the nature of lightning troubles, as will be shown below.

The function $Q_0(r)$ appearing in equation (33) represents the earth

potential at a point due to unit current entering the earth at a distance r from the point; i.e. $Q_0(r)$ is the mutual resistance between two points on the earth's surface separated by the distance r. In the case of a two-layer horizontally stratified earth the function $Q_0(r)$ may be approximated by the following expression:

$$Q_0(r) = \frac{1}{2\pi r}\left[\rho_2 + (\rho_1 - \rho_2)e^{-\gamma_0 r}\right] \tag{47}$$

where

ρ_1 = Resistivity of upper layer, meter-ohms
ρ_2 = Resistivity of lower layer, meter-ohms
$\gamma_0 = k/2d$
d = Depth of upper layer, meters
k = Constant depending on the ratio ρ_1/ρ_2

When r is sufficiently small compared to d, the above expression approaches the limit $Q_0(r) = \rho_1/2\pi r$ and when r is sufficiently large compared to d the expression becomes $Q_0(r) = \rho_2/2\pi r$. Expression (47) gives a fair approximation to the function $Q_0(r)$ as given by curves calculated from rather complicated integrals.[13] Earth resistivity measurements by the so-called "four-electrode measurements" are based on measurements of $Q_0(r)$ and the results of such measurements may usually be approximated to the same degree of accuracy by (47) as by the curves applying accurately for two-layer earth, for the reason that the earth structure usually departs considerably from an ideal two-layer earth. For various ratios ρ_1/ρ_2 the constant k is about as follows:

$\rho_1/\rho_2 =$	100	10	1	.1	.02
$k =$	2	1.84	1.16	.4	.12

Inserting (47) in (33) and proceeding as before, the voltage between core and sheath due to a stroke at the distance y may be written:

$$V(0, y, t) = V(0, a, t)\,\frac{\rho_2\lambda(y) + (\rho_1 - \rho_2)\mu(y)}{\rho_2\lambda(a) + (\rho_1 - \rho_2)\mu(a)} \tag{48}$$

where $V(0, a, t)$ is the voltage for a direct stroke calculated for an equivalent earth-resistivity:

$$\rho_e = \rho_2\lambda(a) + (\rho_1 - \rho_2)\mu(a) \tag{49}$$

and where

$$\lambda(y) = \log\left[(1 + \Gamma y)/\Gamma y\right] \tag{50}$$

$$\mu(y) = \int_0^\infty \frac{1}{r} e^{-\alpha r} e^{-\Gamma x} dx \tag{51}$$

$$\cong \log \frac{1 + y(\gamma_0 + \Gamma)}{y(\gamma_0 + \Gamma)} \quad \text{when} \quad \gamma_0 y \ll 1 \tag{52}$$

$$\cong e^{-\gamma_0 y} \lambda(y) \quad \text{when} \quad \gamma_0 y \gg 1 \tag{53}$$

In applying the above expressions, a rough value of ρ_e is first assumed in calculating $\lambda(a)$ and $u(a)$ and a more accurate value next obtained from

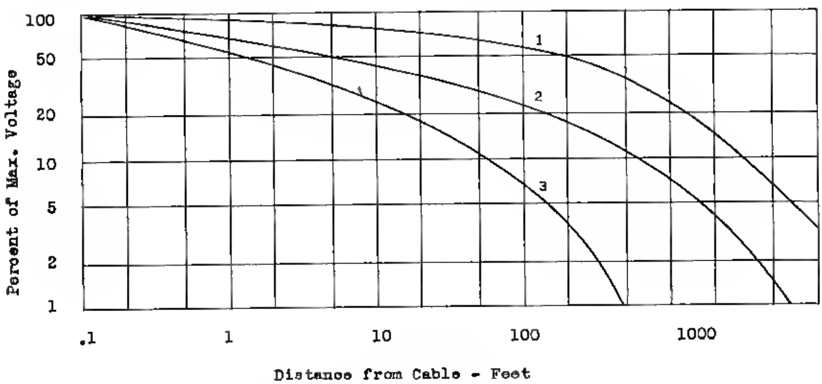

Figure 7—Reduction in voltage between sheath and core with increasing distance from cable to point where current enters ground.

1: Upper layer of 400 meter-ohms and 30 ft. depth. Lower layer of 4000 meter-ohms and infinite depth.

2: Uniformly conducting earth.

3: Upper layer of 1500 meter-ohms and 30 ft. depth.
Lower layer of 150 meter-ohms and infinite depth.

(49). If the value of ρ_e thus obtained differs materially from the assumed value, a second calculation may be required. In the expressions for λ and μ the resistivity ρ_e is to be used in calculating Γ, the latter being taken as $(\nu/2\rho_e t)^{\frac{1}{2}}$ where t is the time to crest value of the voltage as before.

In Fig. 7 is shown the manner in which the voltage decreases with increasing separation for three assumed earth structures. The resistivities and the depth of the upper layer were selected such that the equivalent earth-resistivity, and thus the voltage in the case of a direct stroke, is the same in all cases and equal to 1000 meter-ohms. It will be noticed that in the case where the resistivity of the lower layer is high, the voltage due to a stroke at a distance of 200 ft. is 50% and at a distance of 1000 ft., 25% of the voltage due to a direct stroke. When the cable is small, insulation failures may thus be occasioned by strokes to ground at considerable

distances from the cable, although the resistivity near the surface up to depths of say 50 ft. is only moderately high.

It is seen, however, that wben the earth resistivity of the lower layer is low, failures due to strokes to ground not arcing to the cable are rather unlikely, even when the earth resistivity near the surface is rather high. On account of the higher surface resistivity, however, a greater number of strokes would be expected to arc to the cahle for a given equivalent resistivity, than when the conductivity is uniformly distrihuted. On the other hand, many strokes which would arc to the cable if tbe earth were uniformly conducting may channel through the surface layer to the good conducting lower layer, so that the incidence of direct strokes is reduced on this account. Experience indicates that the latter factor tends to predominate, so that lightning damage is not ordinarily severe when the resistivity is low at depths beyond 20 ft. or so.

In the case of discharges between clouds the coupling between the lightning channel and the cable depends, in the frequency range of importance, to a great extent on the resistance of the lower layer. Thus, when the resistivity of the lower layer is very high the voltages may possihly give rise to insulation failures in the case of small cables, while this is not likely to occur when the resistivity of the lower layer is small or when the earth structure is uniform and of moderately high resistivity.

1.10 *Cables with Insulated Sheaths*

Assume that a short length Δx of insulated sheath is placed on the ground and that a voltage is applied hetween the sheath and a remote ground. When the applied voltage is greater than the hreakdown voltage of the insulation, arcing to ground will take place at numerous equidistant points, provided the insulation and the earth are assumed to he uniform. The voltage between the sheath and adjacent ground increases from zero at a point where arcing takes place to a maximum value midway between two points at which arcing occurs, the maximum value being equal to the breakdown voltage of the insulation. Midway between two arcing points the potential in the earth (referred to infinity) may with negligible error be calculated as though the leakage current through the numerous arcs were uniformly distrihuted along the sheath. This potential in the ground would then he $\Delta I/G\Delta x$, where ΔI is the total leakage current and $1/G\Delta X$ the resistance to ground of the sheath without insulation, G heing the unit length leakage conductance. Midway between the arcing points tbe potential of the sheatb to a remote ground is then:

$$V = V_0 + \Delta I/\Delta x G \tag{54}$$

Let dI_0/dx be the leakage current through the arcs and dI_1/dx the leakage current due to capacity C between the sheath and the adjacent ground. For sinusoidal currents the following equations then hold, when Z is the unit length impedance and G the unit length leakance for a sheath in direct contact with the earth:

$$-\left(\frac{dI_0}{dx} + \frac{dI_1}{dx}\right)\frac{1}{G} + V_0 = V \tag{55}$$

$$-(I_0 + I_1)Z = \frac{dV}{dx} \tag{56}$$

$$-\frac{1}{i\omega C}\frac{dI_1}{dx} = V_0 \tag{57}$$

In the last equation it is assumed that the voltage between sheath and ground is equal to the breakdown voltage of the insulation, although this is not true in the immediate vicinity of the arcs.

Eliminating V the following equation is obtained:

$$\left(\frac{d^2 I_0}{dx^2}\frac{1}{G} - I_0 Z\right) + \left(\frac{d^2 I_1}{dx^2}\frac{1}{Y} - I_1 Z\right) = 0 \tag{58}$$

where:

$$\frac{1}{Y} = \frac{1}{G} + \frac{1}{i\omega C} \quad \text{or:} \quad Y = \frac{i\omega CG}{G + i\omega C}$$

Equation (58) is satisfied when:

$$\begin{aligned} I_0 &= A_0 e^{-\Gamma x} + B_0 e^{\Gamma x} \\ I_1 &= A_1 e^{-\Gamma_1 x} + B_1 e^{\Gamma_1 x} \end{aligned} \tag{59}$$

where Γ and Γ_1 are the propagation constants for a sheath in direct contact with the ground and for an insulated sheath without breakdown, respectively.

$$\Gamma = (GZ)^{\frac{1}{2}}, \qquad \Gamma_1 = (YZ)^{\frac{1}{2}}$$

For a sheath of infinite length the B_0 and B_1 terms vanish, so that:

$$I(x) = I_0 + I_1 = A_0 e^{-\Gamma x} + A_1 e^{-\Gamma_1 x} \tag{60}$$

The constants A_0 and A_1 are obtained from the following boundary conditions:

$$\text{At } x = 0 \qquad I(x) = I(0) = A_0 + A_1 \tag{61}$$

$$\text{As } x \to \infty \qquad I(x) \to A_1 e^{-\Gamma_1 x} = \frac{V_0}{K_1} e^{-\Gamma_1 x} \tag{62}$$

where $K_1 = (Z/Y)^{\frac{1}{2}}$ is the characteristic impedance of the insulated sheath without breakdown.

From (61) and (62)

$$A_1 = \frac{V_0}{K_1} \qquad A_0 = I(0) - \frac{V_0}{K_1} \tag{63}$$

So that:

$$I(x) = I(0)e^{-\Gamma x} + \frac{V_0}{K_1}(e^{-\Gamma_1 x} - e^{-\Gamma x}) \tag{64}$$

For a rubber insulated cable the breakdown voltage V_0 would be in the order of 30,000 volts and the characteristic impedance K_1 would be in the order of 100 ohms. The maximum current which could flow on the sheath without breakdown, V_0/K_1, is then about 300 amperes, while in the case of an average lightning stroke the current $I(0)$ would be about 15,000 amperes (i.e. 30,000 amperes total). The first term in (64) gives the attenuation along a sheath in direct contact with the ground. The current given by this term would diminish from 15,000 amperes to about 2,000 amperes within a distance of $\frac{1}{2}$ mile or so, for a typical lightning stroke wave shape and an earth resistivity of 1000 meter-ohms. At distances of several miles from the stroke point the first term will vanish and the current will be determined by the second term, since $\exp(-\Gamma_1 x)$ will vanish much more slowly than $\exp(-\Gamma x)$.

In the case of a stroke to ground the impressed electric force in the ground along the sheath is

$$E_0(x) = -dV_e(x)/dx \tag{65}$$

where V_e is the earth potential due to the lightning stroke current and is given by

$$V_e(x) = \frac{J\rho}{2\pi(x^2 + y^2)^{\frac{1}{2}}} \tag{66}$$

Instead of equation (58), the following equation is obtained for the currents in the sheath

$$\left(\frac{d^2 I_0}{dx^2}\frac{1}{G} - I_0 Z\right) + \left(\frac{d^2 I_1}{dx^2}\frac{1}{Y} - I_1 Z\right) = -E_0(x) \tag{67}$$

Writing $E_0(x) = cE_0(x) + (1 - c)E_0(x)$, the solution of the latter equation may be written as the sum of two solutions of the form given by (6) and (7). After the constants A_0, B_0 applying to the current I_0 and the constants A_1 and B_1 applying to the current I_1 have been determined from the boundary

conditions, in the same manner as before, the total sheath current may be written in the form:

$$I(x) = cI_g(x) + (1 - c)I_i(x) \tag{68}$$

where I_g is the current for a grounded sheath, I_i the current entering a perfectly insulated sheath by virtue of its capacity to ground and c is given by:

$$c = V_0/V^0 \cong V_0/V_e(0) \tag{69}$$

where V^0 is the potential difference between sheath and adjacent ground without breakdown at $x = 0$, which is substantially equal to the earth potential. The above relationship for the constant c is obtained by applying equation (57) at $x = 0$ to the general solution for I_1, V^0 being given by:

$$V^0 = \frac{G}{G + i\omega c} \int_0^\infty E_0(x)e^{-\Gamma_1 x}\, dx \tag{70}$$

The voltage between core and sheath of an insulated cable may be written in a similar manner when I_g is replaced by V_g, the voltage for a cable in direct contact with the ground, and I_i replaced by V_i, the voltage for a cable insulated from ground.

From (68) and (69) it will be seen that when $V_e(0)$ is much greater than the breakdown voltage of the insulation, the current entering the sheath is nearly the same as for a sheath in direct contact with the ground. Thus, when the earth resistivity is 1000 meter-ohms, and the stroke current 30,000 amperes, the earth-potential at a distance of 30 meters (100 feet) is 160,000 volts. The impulse breakdown of the insulation may be in the order of 30,000 volts, so that the current entering the sheath will be substantially the same as for a cable in direct contact with the soil. When the earth-potential at $x = 0$ is only slightly larger than the breakdown voltage of the insulation, however, the current entering the sheath through punctures in the insulation is fairly small.

In the above derivation the voltages and currents were assumed to vary sinusoidally which, of course, is a rather rough approximation in a phenomenon where breakdown occurs after the voltage reaches a certain instantaneous value. While the derivation is not accurate, it does indicate under what conditions an insulated cable behaves like a cable in direct contact with the ground.

1.11 *Oscillographic Observations of Lightning Voltages*

To obtain data on the characteristics of lightning voltages in buried cable, five magnetic string oscillographs were installed for one lightning

season along a 50-mile section of the Stevens Point-Minneapolis route. The oscillographs, which were arranged to trip where the voltage exceeded 100 volts, recorded lightning voltages due to some 600 strokes on 38 days, the Weather Bureau average heing 33 thunderstorm days for this region in the same months. The character of the voltages varied widely from sharp transients of a few millisecond duration to slowly changing voltages lasting .2 seconds from one zero value to the next, voltages due to multiple discharges heing quite common, the interval hetween voltage peaks in such cases being in the order of .1 second. Of the disturhances, 90% lasted for more than .1 second, 50% for more than .4 and 10% for more than 1.25 second, the maximum duration heing 2.3 seconds. By way of comparison, the ohserved duration of discharges to a tall structure (3) were, in respectively 90, 50 and 10% of all cases, in excess of .08, .3 and .6 second, the maximum duration heing 1.5 second. The maximum voltage recorded was 940 volts and was prohahly due to a stroke to ground near the cahle. Ahout 2% of the voltages were in excess of 500 volts, most of these and the lower voltages heing due to discharges hetween clouds, as indicated by the opposite polarity of the voltages at the two ends of the test section. The wave shape of the voltages at the ends of the section were much the same as at intermediate points, even for the sharpest surges recorded, the attenuation along the core-sheath circuit heing quite small. It is possihle that suhstantially higher voltages than ohserved may ohtain in the case of severe discharges along a path parallel to and directly above the cahle, and that such voltages may produce cahle failure if the core insulation is helow normal.

While the oscillographs were arranged to trip on 100 volts, a smaller voltage was recorded in 40% of all cases, as the peaks were too fast to he recorded hy the type of oscillograph used. It is also possihle that for this reason fast voltage peaks in excess of the maximum given ahove may have escaped measurement.

II. Lightning Trouble Expectancy

2.1 *General*

In estimating the liahility of a cahle to lightning damage, it is assumed helow that once the core insulation is punctured, as it is likely to he at several points, at least one permanent failure will occur. The lightning trouhle expectancy curves presented here thus give the numher of times lightning damage is likely to occur, without consideration of the extent of the damage on each occasion. Each case of lightning damage usually involves several pairs and, hased on experience, repair of each such case would require about four sheath openings. Damage due hoth to direct strokes and strokes to ground is included. Discharges hetween clouds have heen neglected as a

source of lightning damage, however, as the voltages are likely to be insufficient unless the insulation is below normal.

The curves of lightning trouble expectancy calculated here are significant only if troubles on a long cable route are considered over a period of several years, so that the mile-years covered are in the order of 1000 or more.

2.2 *Incidence of Strokes to Ground*

To estimate the lightning trouble expectancy it is necessary to consider the incidence of strokes to ground in the vicinity of the cable, the number

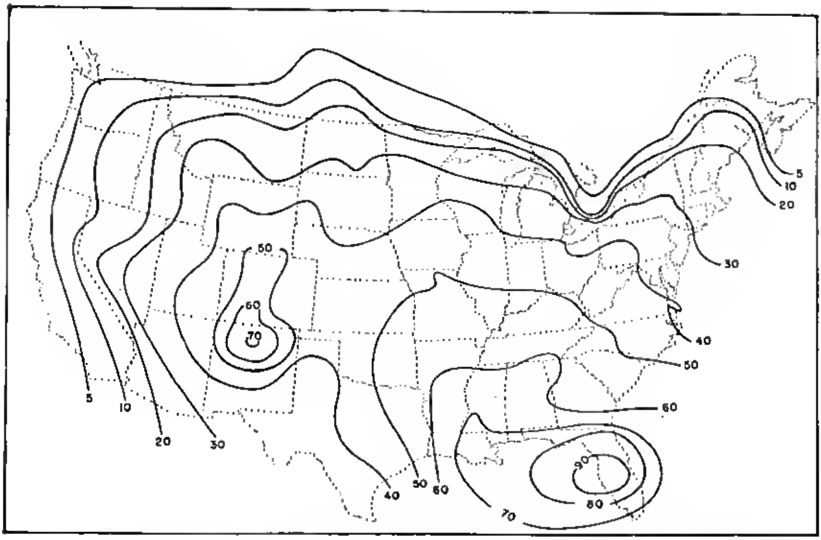

Figure 8—Map showing the average number of thunderstorm days per year.

of such strokes that will arc to the cable and cause damage in this manner and the number that will give rise to failure without arcing to the cable.

Magnetic link measurements (14) indicate that high tension transmission lines will be struck by lightning about 113 times per 100 miles per year, on the average, the minimum incidence in one year being about one half and the maximum about 1.6 times the average value. The above average incidence is based on observations covering about 1600 mile-years and applies for lines traversing areas where some 35 thunderstorm days are expected per year, as indicated by data issued by the U. S. Weather Bureau and collected over a period of 30 years,[15] and shown in Fig. 8. The above data on strokes to transmission lines may be used to estimate the rate of lightning strokes to ground, provided the width of the zone within which a transmission line will attract lightning can be determined.

Based on laboratory observations on small-scale models[16, 17, 18, 19], a line above ground will attract lightning strokes within an average distance on each side of the line which is about 3.5 times the height of the line when the cloud is positive and about 5.5 times the height when the cloud is negative. When the average height of a transmission line ground structure above ground is taken as 70 feet, a 100-mile line will thus attract positive strokes (i.e., strokes originating from a positive cloud) within an area of about 9.3 square miles and negative strokes within an area of approximately 14.5 square miles.

About 15% of the strokes to transmission line ground structures have positive polarity,[4, 5] so that the average rate of positive strokes to ground would be about 1.8 and that of negative strokes about 6.6 per square mile per year. The rate of positive and negative strokes to ground would thus be about 8.4 per square mile per year in areas where the yearly number of thunderstorm days is about 35, corresponding to about 2.4 strokes per square mile per 10 thunderstorm days.

Based on the above date, the ratio of negative to positive strokes to ground in open country would be 3.6. The ratio of negative to positive strokes has been determined by various investigators in different ways.[7] The ratio derived from measurement of field changes during thunderstorms varies between 2.1 and 6.5, that obtained from voltages observed in antennas is about 2.9, while point discharge recorder measurements indicate a ratio of 3.5 and the magnetization of basalt rocks struck by lightning indicates a ratio of 2.25. The above data were obtained in the temperate zone; in the tropics nearly all strokes have negative polarity.

2.3 *Arcing to Cable of Strokes to Ground*

As the stepped leader of a lightning discharge approaches the earth, charges accumulate in the ground under the leader and the resulting flow of current in the ground will give rise to a potential difference between points in the ground under the leader and remote points. Thus, when the tip of the leader has approached within 10 meters off the ground and the leader current is assumed to be as high as 500 amperes and the earth resistivity to be 1000 meter-ohms, a point directly under the leader will have a potential of 8000 volts with respect to a remote ground. The potential gradient along the surface of the ground would, of course, be affected by the presence of a buried cable. The total potential involved is, however, so small that the effect of a buried cable on the path of the leader would be entirely negligible compared to the effect of irregularities in the surface of the earth. As the tip of the leader contacts the ground, the potential may be large enough so that the leader may arc to a cable located within 2 ft. or so of the leader. Only

after the return stroke is initiated, however, is the potential large enough so that arcing will occur for appreciable distances.

When a stroke current J enters the ground, the electric force in the ground at the distance r from the stroke point is given by

$$E(r) = J\rho/2\pi r^2 \tag{71}$$

ρ being the earth resistivity. If the breakdown voltage gradient of the soil is e_0, breakdown of the soil will take place until $E(r) = e_0$, or for a distance:

$$r_0 = (J\rho/2\pi e_0)^{\frac{1}{2}} \tag{72}$$

For a distance r_0 around the stroke point the soil may then be regarded as a conductor of negligible resistivity.

The resistance encountered by the lightning channel in the ground is then

$$R_0 = \frac{\rho}{2\pi r_0} = \left(\frac{\rho e_0}{2\pi J}\right)^{\frac{1}{2}} \tag{73}$$

Measurements of the surge characteristics of grounds of fairly small dimensions[20, 21] indicate that the breakdown voltage of the soil may vary from roughly 1000 volts per cm to some 5000 volts/cm. The data are, however, quite limited and there is no assurance that the above values represent the limits. In the first of the papers referred to, measurement was made of the resistance encountered when a current of 10,000 amperes crest value was discharged into the ground from an electrode suspended above the ground. The measured resistance is in satisfactory agreement with that obtained from (73) using the earth resistivity and breakdown voltage determined from other measurements in the paper referred to. In connection with the present study some small scale measurements were made of the breakdown voltage of sand between plane electrodes, and of the resistance encountered in discharges into the sand over point electrodes. These measurements indicated a breakdown voltage of 5000 volts per cm for dry sand having a resistivity of 3700 meter-ohms and 2400 volts/cm when sufficient water was added to reduce the resistivity to 100 meter-ohms. The measured resistance of point electrodes was a satisfactory agreement with that calculated from (73) on the basis of the measured resistivities and breakdown voltages. These experiments were made with currents having crest values from 1 to 50 amperes.

Measurements of the breakdown voltage of various types of soil between completely buried spherical electrodes indicate substantially higher voltages than those given above, from about 11 to 23 kv/cm.[22] It is possible that for surface electrodes, breakdown at the lower voltage gradients occurs along the surface of the ground, so as to form a conducting plane of radius

r_0. This circumstance would change the preceding formulas only to the extent that 2π would be replaced by 4. For a given e_0, the resistance would then be about 25% higher.

The radius r_0 is not necessarily the same as the distance across which a lightning stroke would arc to a cable in the vicinity. Streamers will extend in various directions beyond r_0 so that ionization of the soil increases the conductivity for a greater distance, r_0 being the effective radius of an equivalent hemisphere of infinite conductivity.

The potential difference between the conducting sphere of radius r_0 and a point at the distance r_1 is

$$V_{01} = \frac{J\rho}{2\pi}\left(\frac{1}{r_0} - \frac{1}{r_1}\right) = \frac{J\rho}{2\pi}\frac{r_1 - r_0}{r_1 r_0} \tag{74}$$

Let it be assumed that streamers extend beyond r_0 until the average voltage gradient between r_0 and r_1 equals e_1. The potential difference is then:

$$V_{01} = e_1(r_1 - r_0) \tag{75}$$

From (74) and (75):

$$r_1 = \frac{J\rho}{2\pi r_0 e_1} = r_0 \frac{e_0}{e_1} \tag{76}$$

The latter expression applies when the field is assumed to have a radial symmetry about the lightning channel. When a buried cable is present, however, this symmetry is disturbed. Calculations indicate that the potential of the cable at the point nearest to the lightning stroke will be less than 20% of the earth potential at the same point if the cable were absent. As a first approximation the cable may, therefore, be considered to have zero potential. The potential difference between the sphere considered above and the cable is then:

$$V_{02} = \frac{J\rho}{2\pi}\frac{1}{r_0} \tag{77}$$

If r_2 is the distance from the channel to the cable, the potential difference is also given by:

$$V_{02} = e_1(r_2 - r_0) \tag{78}$$

From (77) and (78):

$$r_2 = r_0\left(1 + \frac{e_0}{e_1}\right) \tag{79}$$

The arcing distance calculated in this manner will be the maximum, while that calculated from (76) will be the minimum distance.

From measurements made of the effective corona radius of a conductor in air,[22] it is found that the latter may be determined on the assumption that

breakdown occurs until $e_0 \cong 14000$ volts/cm. On the other hand it is known that arcing between two conductors a considerable distance apart will occur when the average gradient is about 10,000 volts/cm. For air the ratio e_0/e_1 is thus about 1.4. For breakdown in the air, the ionization need not be very dense in order that the corona envelope may be regarded as conducting as regards displacement currents. For breakdown in the earth, however, the ionization must be much more complete in order to provide substantially better conductivity than the soil. The ratio e_0/e_1 is, therefore, likely to be greater, and has been taken as 2 in the following.

If the latter ratio is assumed for soil, it is seen by comparison of (76) and (79) that the presence of a cable may increase the arcing distance as much as 50%.

The arcing distance may be written in the form

$$r_1 = (J\rho)^{\frac{1}{2}} q_1, \qquad r_2 = (J\rho)^{\frac{1}{2}} q_2 \tag{80}$$

where:

$$q_1 = \left(\frac{1}{2\pi e_0}\right)^{\frac{1}{2}} \frac{e_0}{e_1}, \qquad q_2 = \left(\frac{1}{2\pi e_0}\right)^{\frac{1}{2}} \left(1 + \frac{e_0}{e_1}\right) \tag{81}$$

With $e_0/e_1 = 2$ the following values are obtained

$e_0 =$	250,000	500,000 volts/meter
$q_1 =$	$1.6 \cdot 10^{-3}$	$1.13 \cdot 10^{-3}$
$q_2 =$	$2.4 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$

Values of q_1 and q_2 about 25% greater than those given above are obtained by assuming that breakdown does not take place in the soil, but that a conducting plane of radius r_0 is formed by ionization of the air near the ground. Expression (71) is in this case replaced by $E(r) = J\rho/4r^2$ and the expressions for q become

$$\begin{aligned} q_1 &= \left(\frac{1}{4e_0}\right)^{\frac{1}{2}} \frac{e_0}{e_1} \\ q_2 &= \left(\frac{1}{4e_0}\right)^{\frac{1}{2}} \left(1 + \frac{e_0}{e_1}\right) \end{aligned} \tag{82}$$

In the following $q = 2.5 \cdot 10^{-3}$ has been taken as representative for low-resistivity soil and $q = 1.5 \cdot 10^{-3}$ for high-resistivity soil. When the current is expressed in kiloamperes, the corresponding values are .08 and .047.

The distance to which a stroke may arc is accordingly taken as:

$\rho \leq 100$ meter-ohms	$\rho \geq 1000$ meter-ohms
$r = .08\ (J\rho)^{\frac{1}{2}}$ meter	$r = .047\ (J\rho)^{\frac{1}{2}}$ meter
$= .26\ (J\rho)^{\frac{1}{2}}$ feet	$= .15\ (J\rho)^{\frac{1}{2}}$ feet

where J is in kiloamperes.

These values are, of course, of an approximate nature, and are only indicative of what may be expected under average conditions. In some cases the breakdown voltage of high-resistivity soil may be substantially lower than assumed, while that of low-resistivity soil may be noticeably higber.

2.4 *Crest Current Distribution for Strokes to Ground*

When the earth resistivity is taken as high as 5000 meter-ohms and the breakdown voltage of the soil is taken as high as 5000 volts/cm, the resistance encountered by the channel on the ground for a current of 25,000 amperes is about 250 ohms. If the lightning channel were a long conductor already in existence at the initiation of the return stroke and capable of carrying the stroke current without being fused, the current would be propagated upward with the velocity of light and the surge impedance of the channel would be in the order of 500 ohms. Due to the resistance in the ground the current would then be some 30% smaller than for a stroke to an object of zero resistance to ground. However, the ligbtning cbannel may not be regarded in the above manner, but as a conductor which is gradually prolonged at about 1/10 the velocity of light, and the impedance of the channel is then much larger, perbaps 5000 ohms. The surge impedance of a long insulated conductor having unit length capacitance C is $(1/Cv)$, v being the velocity of propagation. When energy is required to create the conductor, so that the velocity of propagation is reduced, tbe impedance is increased. Because of the higb impedance of the channel, the resistance encountered in the ground may, therefore, be neglected as regards the effect on the crest current. The crest current distribution curve for strokes to transmission line ground structures may thus be used also in the case of strokes to ground, although a different distribution curve is obtained for those of the strokes to ground which arc to buried cable (Section 2.7).

2.5 *Failures Due to Direct Strokes and Strokes to Ground*

In calculating the number of failures due to direct strokes and strokes to ground, the earth is assumed to be a plane surface. A tree placed at random may attract a lightning stroke toward a cable or it may divert it from the cable and tbe net effect of a large number of trees along a route of substantial length is likely to be small. This is also true for variations in the terrain.

When N is the number of ligbtning strokes to ground per unit of area, and s the length of the cable, the number of lightning strokes on both sides of the cable within y and $y + dy$ is:

$$dN = 2Ns\, dy \tag{83}$$

A lightning stroke at the distance y will cause cable failure when the crest current i exceeds a certain value which depends on the distance:

$$i = f(y) \tag{84}$$

The fraction of all lightning strokes which has a crest current larger than i will be designated $P_0(i)$. The fraction of the lightning strokes dN which will cause cable failures is then:

$$dn = dNP_0(i) = 2NsP_0(i)\,dy \tag{85}$$

The number of cable failures along the length s due to all lightning strokes to ground up to the maximum distance Y that need to be considered is:

$$n = 2Ns\int_0^Y P_0(i)\,dy \tag{86}$$

For the purpose of computation it is convenient to change the variable in the latter integral from y to i. With $y = f^{-1}(i) = y(i)$, $di = dy \cdot f^1(y)$, $i_0 = f(0)$ and $I = f(Y)$, the following integral is obtained:

$$n = 2Ns\left[YP_0(i) - \int_{i_0}^{I} y(i)P_0'(i)\,di\right] \tag{87}$$

In (87), I is the maximum stroke current that needs to be considered and may actually be replaced by infinity, as will be evident later on. The current i_0, which is the minimum current that will cause insulation puncture in the case of a direct stroke, may readily be determined from the breakdown voltage of the insulation and the calculated voltage between core and sheath per kiloampere, in the manner illustrated in Section 2.4.

In order to evaluate the integral of (87), it is divided as follows:

$$n = 2Ns\left[YP_0(I) - \int_{i_0}^{i_1} y_1(i)P_0'(i)\,di - \int_{i_1}^{I} y_2(i)P_0'(i)\,di\right] \tag{88}$$

When $i < i_1$, failures of the cables will be due to arcing of the stroke to the cable and when $i > i_1$, failures will occur before arcing takes place, due to the leakage current entering the sheath. Within each of the above two ranges the relationship of y to i is different and is designated $y_1(i)$ and $y_2(i)$, respectively.

As already shown, failures due to arcing will take place when:

$$y_1(i) \leq q(\rho i)^{\frac{1}{2}} \tag{89}$$

where q is defined as before and ρ is the earth resistivity in meter-ohms.

In Section 1.7 it was shown that failures due to leakage current will occur when

$$i \geq i_0/\lambda(y)$$

wbere

$$\lambda(y) = \log\left(\frac{1 + \Gamma y}{\Gamma y}\right) \Big/ \log(1/\Gamma a)$$

Γ being the propagation constant of the sheath-eartb circuit and a the radius of the sheath. The solution of the latter equation for y is:

$$y = y_2(i) = \frac{1}{\Gamma(e^{mi_0/i} - 1)} \cong \left(\frac{2\rho t}{\nu}\right)^{\frac{1}{2}} \frac{1}{e^{mi_0/i} - 1} \tag{90}$$

where

$m = \log(1/a\Gamma)$
$\Gamma \cong (\nu/2\rho t)^{\frac{1}{2}}$ per meter
$\nu = 1.256 \cdot 10^{-6}$ henries per meter
$t \cong 10^{-4}$ sec. $\cong$ time to crest of core-sheath voltage.

When (89) and (90) are equated, the following expression is obtained:

$$i^{\frac{1}{2}}(e^{mi_0/i} - 1) = \left(\frac{2t}{\nu q^2}\right)^{\frac{1}{2}} \tag{91}$$

The value of i which satisfies the latter equation is the value i_1 defined above, and is shown in Fig. 9 as a function of mi_0 for various values of $(2t/\nu q^2)^{\frac{1}{2}}$.

When (89) and (90) are inserted in (88), the latter integral may be expressed as follows:

$$n = 2Ns\rho^{\frac{1}{2}}\left(q[H(i_0) - H(i_1)] + \left(\frac{2t}{\nu}\right)^{\frac{1}{2}} G(i_1, mi_0)\right) \tag{92}$$

where the current is in kiloamperes and:
N = Number of strokes to ground per square meter
s = Length of cable in meters
$q \cong .08$ when $\rho \leq 100$, $q \cong .047$ when $\rho \geq 1000$.

$$H(i) = -\int_i^I i^{\frac{1}{2}} P_0'(1)\, di \tag{93}$$

$$G(i, mi_0) = \frac{P_0(I)}{e^{mi_0/I} - 1} - \int_i^I \frac{P_0'(i)\, di}{e^{mi_0/i} - 1} \tag{94}$$

The first term in (94) equals $YP_0(I) = y_2(I)P_0(I)$. Since $P_0'(i)$ is negative, the above integrals will have positive values.

The term $q[H(i_0) - H(i_1)]$ of (92) gives the portion of failures due to direct strokes while the term involving the function G gives the portion of failures due to ground strokes not necessarily arcing to the cable altbough

they may do so (i.e. many of the currents in excess of i_1 may arc to the cable, although this is not essential in order to produce cable failure).

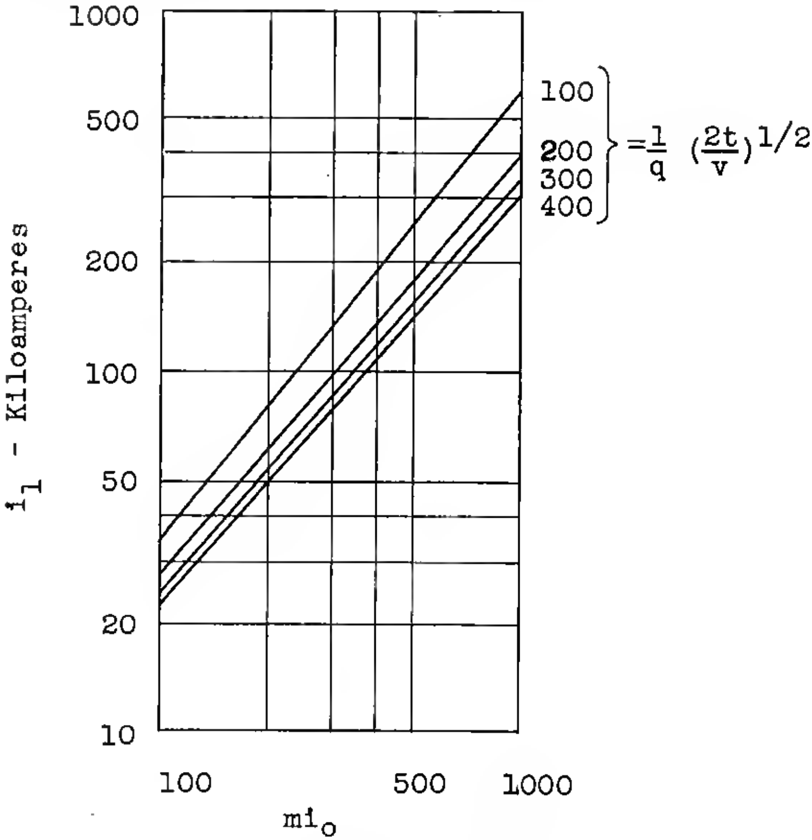

Figure 9—Solution of the equation:

$$i_1^{\frac{1}{2}}(e^{mi_0/i_1} - 1) = \frac{1}{q}\left(\frac{2t}{v}\right)^{\frac{1}{2}}$$

If strokes to ground not arcing to the cable were neglected as a source of failures, the number of failures would equal:

$$n_d = 2Ns\rho^{\frac{1}{2}}qH(i_0) \tag{95}$$

If, on the other hand, the dielectric strength of the earth were assumed to be infinite, so that none of the strokes to ground would arc to the cable, the number of failures would equal

$$n_g = 2Ns\rho^{\frac{1}{2}}\left(\frac{2t}{v}\right)^{\frac{1}{2}} G(i_1, mi_0) \tag{96}$$

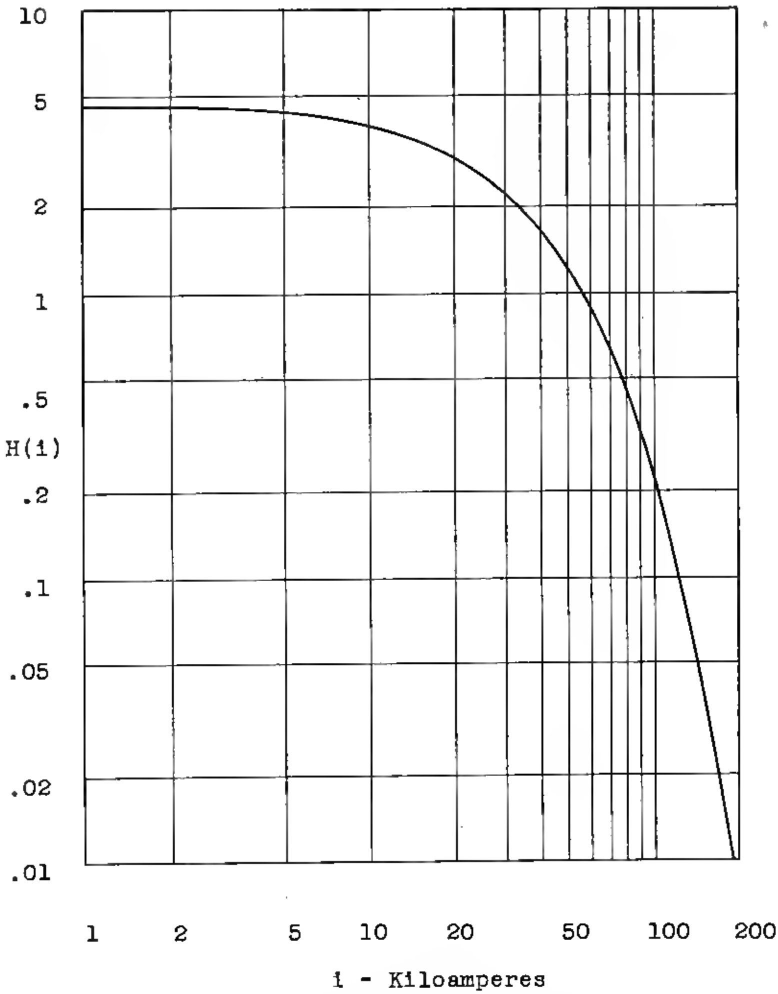

Figure 10—Function $H(i)$ when $P_0(i)$ is approximated by:
$P_0(i) = \exp(-ki)$, $k = .038$ per kiloampere.
$H(i) = k^{-\frac{1}{2}}[(ki)^{\frac{1}{2}}e^{-ki} + \frac{1}{2}\pi^{\frac{1}{2}} \operatorname{erfc}(ki)^{\frac{1}{2}}]$

From Fig. 1, it is seen that $P_0(i)$, as represented by curve 1, is nearly a straight line on semi-log paper and may, therefore, be approximated by:

$$P_0(i) \cong e^{-ki} \tag{97}$$

With $k = .038$ per kiloampere, a straight line is obtained which coincides with curve 1 at $i = 0$ and $i = 100$ kiloamperes, and this value of k has been used in the following. The functions H and G obtained with this approximation are shown in Figs. 10 and 11. In obtaining these integrals, the up-

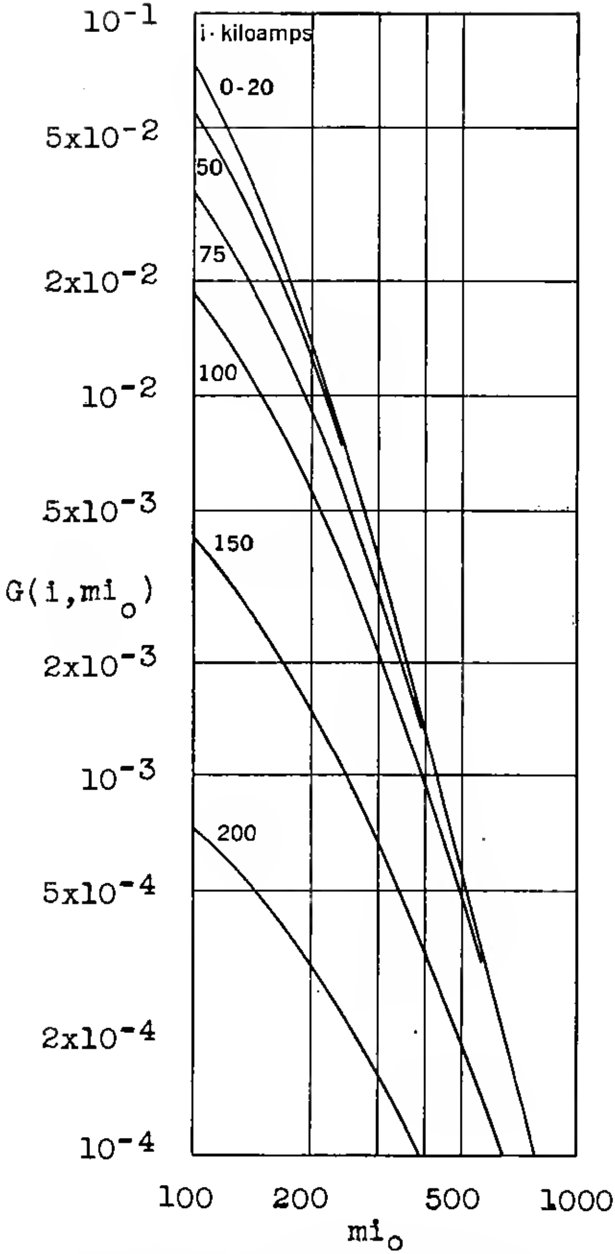

Figure 11—Function $G(i, mi_0)$ when $P_0(i)$ is approximated by:
$P_0(i) = \exp(-ki)$; $k = .038$ per kiloampere.

$$G(i,\ mi_0) = k \int_i^{\infty} \frac{e^{-ki}\ di}{e^{mi_0/i} - 1}$$

per limit I may be replaced by infinity, and it is also seen that the first term in (94) then vanishes.

The lightning trouble expectancy as calculated from (92) is shown in Fig. 12 as a function of the earth resistivity for various sheath resistances. The curves are based on 2.4 strokes to ground per square mile, which is approximately the number of strokes per square mile during 10 thunderstorm days. The number of thunderstorm days per year along a given route is obtained from Fig. 8 and thus the number of times lightning failures would be expected during one year.

2.6 *Expectancy of Direct Strokes*

The incidence of direct strokes to the cable may be obtained from (96) with $i_0 = 0$ kiloamperes.

The number of strokes arcing to the cable is thus

$$n_a = 2Ns\rho^{\frac{1}{2}}qH(0) \tag{98}$$

The cable will thus attract strokes within an effective distance.

$$y = \rho^{\frac{1}{2}}qH(0) \tag{99}$$

$\rho \leq 100$ meter-ohms	$\rho \geq 1000$ meter-ohms
$y = .365\ \rho^{\frac{1}{2}}$ meters	$y = .22\ \rho^{\frac{1}{2}}$ meters
$= 1.2\ \ \rho^{\frac{1}{2}}$ feet	$= .7\ \ \rho^{\frac{1}{2}}$ feet

2.7 *Crest Current Distribution for Direct Strokes*

The fraction of the strokes to the cable having crest values in excess of i is given by:

$$\begin{aligned} P(i) &= H(i)/H(0) \\ &= 2\left(\frac{ik}{\pi}\right)^{\frac{1}{2}} e^{-ik} + \operatorname{erfc}\,(ik)^{\frac{1}{2}} \end{aligned} \tag{100}$$

and is shown by curve 2 in Fig. 1. It will be noticed that a buried cable attracts a greater proportion of heavy currents than a transmission line, because of the circumstance that heavy currents to ground arc for greater distances.

2.8 *Lightning Trouble Experience*

As mentioned before, lightning damage may be due to denting or to fusing of holes in the sheath, or to excessive voltages between the sheath and the cable conductors. Only the latter form of lightning failures have been considered here, since they predominate for cable of the size now being used, particularly in high-resistivity areas, and are likely to extend for a considerable distance to both sides of the point struck by lightning and are

thus more difficult to repair. For full-size cable in low-resistivity areas, however, insulation failures are more likely to occur as a result of sheath denting. For instance, along the 300-mile Kansas City-Dallas full-size cable route, where the earth resistivity is in the order of 100 meter-ohms, and where there are some 50 thunderstorm days per year, failures over a period of about 15 years have occurred about .5 times per 100 miles per year. Of these troubles 85% were due to sheath denting as a result of arcing between the tape armor and the sheath. Based on (99), 100 miles of cable would attract lightning strokes within an area of .5 square mile, so that the cable would be struck about six times per 100 miles per year, when the number of strokes per square mile per year is 2.4 per 10 thunderstorm days (Section 2.2). The rate of lightning failures experienced on this route may thus be accounted for by assuming that about 7% of the strokes, i.e. currents in excess of 90 kiloamperes as obtained from curve 2, Fig. 1, will produce sheath denting severe enough to cause insulation failure, while about 1%, i.e. currents in excess of 140 kiloamperes, will cause insulation failure due to excessive voltage. The latter value is in substantial agreement with that calculated for a full-size cable when the earth resistivity is assumed to be 100 meter-ohms. It is evident from the above examples that in low-resistivity areas lightning troubles will not be a problem, and this is also borne out by experience on other routes installed in such territory during the last few years.

All cable installed in high-resistivity territory since 1942 has been provided with doubled core insulation and with shield wires, in spite of which considerable damage has been experienced on some routes, as between Atlanta and Macon. This appears to have been due partly to the circumstance that in many cases the insulation in splices and accessories has not been equal to that obtained in the cable through the use of extra core wrap, and that in some cases damage has been due to holes fused in the sheath due to arcing between the sheath and the shield wires. As an example, along the Atlanta-Macon route there are some 70 thunderstorm days per year and the average effective earth resistivity is about 1300 meter-ohms. The corresponding estimated rate of direct strokes to the cable is about 20 per 100 miles per year. It is estimated, in the manner outlined in section 3.0, that only stroke currents in excess of 80 kiloamperes are likely to damage the cable, so that on the basis of curve 2, Fig. 1, cable failures would be expected to occur about 2 times per 100 miles per year. The actual rate of trouble experienced on this route over two years has been about five times higher, so that some 50% of the strokes to the cable; i.e., currents in excess of 30 kiloamperes or so, appear to have caused cable failures, most of which occurred in splices and accessories.

It is evident from the above examples that careful examinations of trouble records are required before the observed rate of lightning failures can be adequately compared with that obtained from theoretical expectancy curves. If the cable as well as splices and accessories actually have a dielectric strength as assumed in the calculations, it is likely that the average rate of failures due to excessive voltages experienced over a long period will not be any greater than estimated from these curves.

Based on experience, an average of 4 sheath openings is required to repair damage caused by excessive voltage between the sheath and the cable conductors, as compared to about 2 sheath openings when the damage is due mainly to denting and fusing of the sheath, as in the case of full-size, tape-armored cable in low-resistivity territory. Although the damage may be confined to one point, it cannot usually be located by a single sheath opening.

III. Remedial Measures

3.1 *General*

From Fig. 12 it is evident that the rate of cable failures to be expected, and hence the need for remedial measures, depend greatly on the earth resistivity. Experience has indicated that lightning damage is likely to be encountered even when the surface resistivity is fairly low, provided the resistivity beyond depths of 10 or 20 ft. or so is very high. Considerably less trouble has been experienced where the resistivity below this depth is low, even where the surface resistivity has been high. The lightning stroke may then channel through the surface layer to the good conducting lower layer, so that direct strokes are not experienced as frequently in spite of the high surface resistivity. As a guide in applying protective measures, earth resistivity measurements are usually made along new cable routes.[23]

The curves given in Fig. 12 may also be used to find the lightning trouble expectancy when extra core insulation, shield wires or both are used. Thus when the insulation strength is doubled the effect is the same as if the sheath resistance is halved. If the shield wires reduce the voltage by a shield factor η, the effect is the same as if the sheath resistance is multiplied by η. Considering direct strokes only, curve 2 in Fig. 1 may be used to find the percentage reduction in lightning strokes that will damage the cable, when the stroke current which the cable is able to withstand is increased by extra insulation or shield wires.

3.2 *Extra Core Insulation*

One method of reducing failures caused by lightning strokes to buried cables is to increase the insulation between the cable conductors and the

sheath, no extra insulation being required between individual cable conductors. This has already been done for most new installations. The cable itself, cable stubs, loading cases, and gas alarm contactor terminals are all provided with sufficient extra insulation to douhle the dielectric strength hetween cahle conductors and sheath. For a cable like that on which the measurements referred to before were made, such a measure would increase the stroke current which would damage the cable from 30,000 to 60,0000 amperes and would reduce the number of direct lightning strokes that could cause failure by direct arcing to the sheatb to about 20

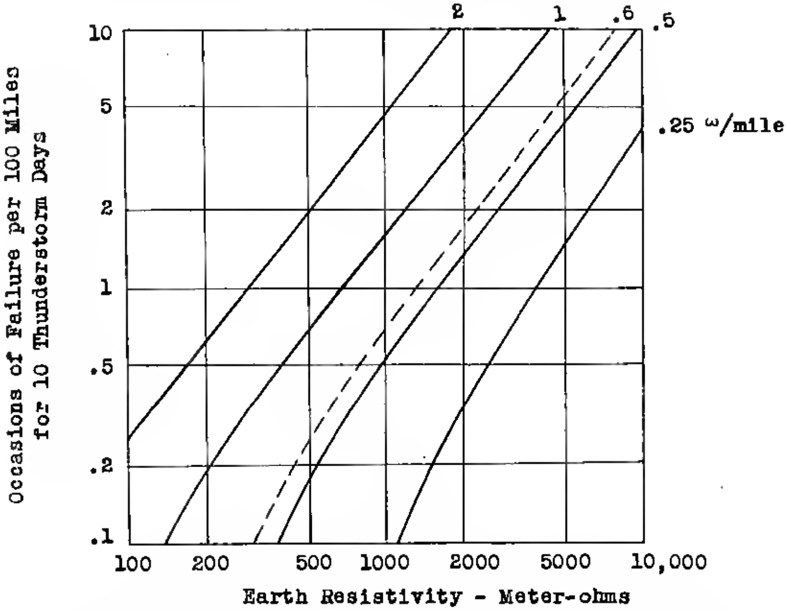

Figure 12—Theoretical lightning trouble expectancy curves showing number of times insulation failures due to excessive voltages would be expected per 100 miles for 10 thunderstorm days, for cables having sheath resistances as indicated on curves. Dashed line represents full-size cable.

per cent of the total instead of 50 per cent (see Fig. 1, curve 2). The numher of failures due to direct strokes would thus have heen reduced 2.5 times.

3.3 *Shield Wires*

Another method, employed in addition to the extra insulation where excessive lightning damage would otherwise be expected, is to bury shield wires over the cable. These conduct away part of the lightning current and thus reduce the amount that flows along the sheath. These wires may be plowed in with the cable, as has been done on several new routes, or may be installed afterward. When the wires and cables are plowed in

together, the arrangement shown in Fig. 13 has been used. The percentage of the current carried hy the wires depends to a greater extent on their inductance relative to that of the sheath, than on their resistance. Two wires are employed, rather than a single wire of smaller resistance, in order to obtain a lower inductance than with a single wire.

On the route hetween Stevens Point and Minneapolis, where the shield wires were installed after the cahle was in place, two 165-mil wires ahout twelve inches apart were plowed in some ten inches above the cahle for a distance of eighty miles. Surge measurements made after these wires were installed indicated that the wires reduced the voltage hetween sheatb and core conductors ahout 60 per cent, in substantial agreement with theoreti-

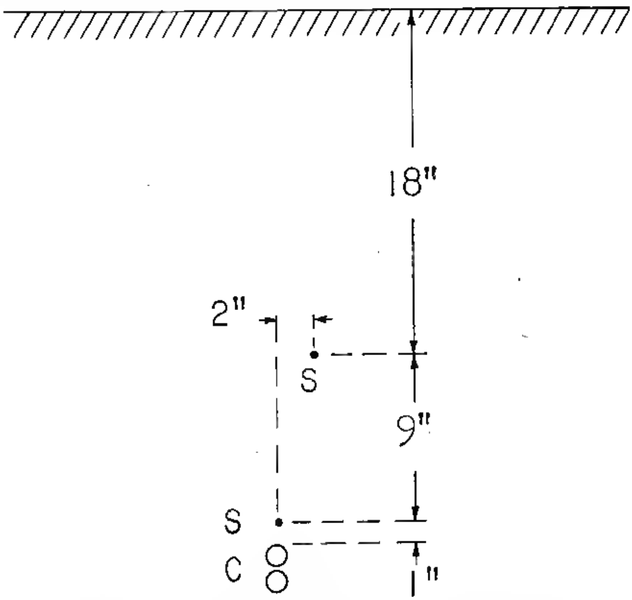

Figure 13—Position of shield wires, *S*, when they are plowed in with cable, *C*.

cal expectations. The cable would then withstand 75,000 amperes rather than 30,000 without shield wires. If a cahle of normal construction were used on the ahove route, the shield wires would thus reduce the number of direct lightning strokes that would be expected to cause insulation failure about 3.3 times, from 50 per cent of the total without shield wires to about 15 per cent with shield wires (see Fig. 1, curve 2). If the insulation strength is only 1000 volts, however, about 80 per cent of all strokes would he expected to cause failure without shield wires and 40 per cent with shield wires, so that the reduction would be suhstantially smaller, as actually appears to be the case on the above route.

Aside from reducing core insulation failures, shield wires also minimize

damage to the sheath covering in the case of strokes to ground near the cable, particularly in the case of thermoplastic or rubber-covered cables. As mentioned in section 1.10, considerable current may enter the sheath of an insulated cable in the case of strokes to ground near the cable, due to numerous punctures in the insulation of the sheath. If conditions along the cable were uniform, the current through each puncture would be so small that the insulation would not be damaged. Due to variations in the resistivity of the soil and the presence of buried metallic structures, however, concentrated arcing may occur, and the insulation may then be damaged in spots, so that corrosion may be initiated. Shield wires reduce the voltage between the sheath and ground and thus the likelihood of damage to the sheath insulation.

It can be demonstrated theoretically, and has been proved by measurements, that when current enters shield wires next to a buried cable in good contact with the earth, the current in the sheath and the voltage between sheath and cable conductors is negligible. The reason for this is that current induced in the sheath by the shield wire current is equal and opposite to the current entering the sheath by virtue of leakage through the ground. Negligible voltages between sheath and core conductors would thus be obtained if shield wires were installed at such a distance that lightning strokes would be intercepted and direct strokes to the cable prevented. To prevent arcing to the cable of strokes to the shield wires, the separation between cable and shield wires would have to be at least 6 feet when the earth resistivity is 1000 meter-ohms, and greater for higher resistivities. Such wires cannot, therefore, be as easily installed in one plowing operation with the cable as shield wires at a smaller spacing. They are, therefore, not considered here, although they have been installed in one instance in a fairly short section where repeated lightning damage had been experienced.

When the sheath, as well as the shield wires, is in intimate contact with the earth, the propagation constant for the shield wires is the same as that for the sheath. In the case of a direct stroke to the cable or the shield wires, the voltage between sheath and shield wires will be so large that they will be in contact with each other at the stroke point by virtue of arcing. The current in the sheath is then:

$$I(x) = \frac{J}{2} \frac{Z_{22} - Z_{12}}{Z_{11} + Z_{22} - 2Z_{12}} e^{-\Gamma x} \tag{101}$$

where J is the total current at $x = 0$, and

$Z_{11} = R_1 + i\omega L_{11}$ = Unit length impedance of sheath
$Z_{22} = R_2 + i\omega L_{22}$ = Unit length impedance of shield wires
$Z_{12} = i\omega L_{12}$ = Unit length mutual impedance of sheath and shield wires

The voltage between sheath and core conductors at $x = 0$ then becomes:

$$
\begin{aligned}
V(0) &= \frac{J}{2} \frac{R}{\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}} \frac{Z_{22} - Z_{12}}{Z_{11} + Z_{22} - 2Z_{12}} \left(\frac{1}{i\omega}\right)^{\frac{1}{2}} \\
&= \frac{J}{2} \frac{R}{\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}} \eta \left(\frac{1}{i\omega}\right)^{\frac{1}{2}} \frac{p + \alpha_0}{p + \beta_0}
\end{aligned}
\tag{102}
$$

where

$$\eta = (L_{22} - L_{12})/(L_{11} + L_{22} - 2L_{12}) \tag{103}$$

$$\alpha_0 = R_2/(L_{22} - L_{12}) \tag{104}$$

$$\beta_0 = (R_1 + R_2)/(L_{11} + L_{22} - 2L_{12}) \tag{105}$$

The function S' is in this case

$$S'(0, t) = \frac{J}{2} \frac{R}{\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}} \eta \left[\left(\frac{1}{\pi t}\right)^{\frac{1}{2}} + \frac{\alpha_0 - \beta_0}{\beta_0^{\frac{1}{2}}} h(\beta_0^{\frac{1}{2}} t^{\frac{1}{2}})\right] \tag{106}$$

where the function h is defined as before.

When the shield wires are at a sufficient distance from the sheath, so that proximity effects may be neglected, the self and mutual inductances are as follows:

$$L_{11} - L_{12} = \frac{\nu}{2\pi} \log \frac{r_{12}}{r_{11}} \tag{107}$$

$$L_{22} - L_{12} = \frac{\nu}{2\pi} \log \frac{r_{12}}{r_{22}} \tag{108}$$

where

$\log = \log_\epsilon$ and:

$\nu = 1.256 \cdot 10^{-6}$ henries per meter

r_{11} = Radius of sheath

r_{22} = Radius of shield wire

r_{12} = Distance between sheath and shield wire.

With more than one shield wire, r_{22} is the geometric mean radius and r_{12} their geometric mean separation from the sheath. When there are two cables, as is frequently the case, r_{11} is the geometric mean radius of the cables and R is their combined sheath resistance.

The surge voltage obtained by solution of (1), for a current as given by (19), is in this case:

$$
\begin{aligned}
V(0, t) = \frac{IR\eta}{2(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}})} \Bigg[&\frac{a - \alpha_0}{a - \beta_0} a^{\frac{1}{2}} h(a^{\frac{1}{2}} t^{\frac{1}{2}}) \\
&- \frac{b - \alpha_0}{b - \beta_0} b^{-\frac{1}{2}} h(b^{\frac{1}{2}} t^{\frac{1}{2}}) + \frac{(\alpha_0 - \beta_0)(b - a)}{(a - \beta_0)(b - \beta_0)} \beta_0^{-\frac{1}{2}} h(\beta_0^{\frac{1}{2}} t^{\frac{1}{2}}) \Bigg]
\end{aligned}
\tag{109}
$$

When $\alpha_0 = \beta_0$, the voltage with shield wires differs from that obtained without shield wires by the factor η. With two 165-mil wires 12 inches apart and 10 inches above the cable considered before, $\alpha_0 = 1.3 \cdot 10^3$, $\beta_0 = 1.5 \cdot 10^3$ and $\eta = .47$. In this case α_0 differs only slightly from β_0 so that the voltage is reduced by the factor η. Measurements made before and after the shield wires were installed indicated a reduction factor of .40 (i.e. the voltage with was .4 times the voltage without shield wires). The reasons for the smaller observed factor is partly that the shield wires are in more intimate contact with the earth than the sheath, and partly that the resistivity of the soil above the cable, where the shield wires are located, is somewhat smaller than the resistivity at the depth of the cable.

With 104-mil wires, $\alpha_0 = 3.2 \cdot 10^3$, $\beta_0 = 2.3 \cdot 10^3$ and $\eta = .49$. When the reduction factor is determined more accurately by calculating the crest voltage with shield wires from (109) and comparing it with the crest voltage without shield wires, a value of .52 is obtained as compared with .47 for 165-mil wires. It is thus seen that, within certain limits, the voltage reduction provided by shield wires depends to a comparatively small extent on the size of the wires, the resistance of 104-mil wires being about 2.5 times that of 165-mil wires.

3.4 *Lightning-Resistant Cable*

As mentioned before, buried cable may be covered by jute, thermoplastic, or rubber for protection against corrosion. The coating may be damaged by gophers or by lightning, and severe corrosion may be experienced at points where the coating is ruptured, particularly when thermoplastic or rubber coating is used. Even when the earth resistivity is low and protection against core insulation failures due to excessive voltage would not be required, the sheath coating may be damaged rather frequently.

Shield wires may effect a substantial reduction in core insulation failures and may also prevent damage to the coating in the case of strokes to ground at some distance from the cable. In the case of direct strokes, however, arcing between the shield wires and the sheath will damage the sheath coating and may also fuse a hole in the sheath, although there may be no insulation failures due to excessive voltage between the sheath and the cable conductors.

Reduction of damage to the coating and to the sheath occasioned by lightning, rodents, or corrosion and protection against core insulation failures occasioned by excessive voltage or crushing of the sheath may be secured by providing the sheath with a thermoplastic or rubber coating and an outside copper shield. If various auxiliary equipment connected to the sheath, such as load coils, gas pressure contactors and terminals are also properly

insulated from ground, currents will not then enter the sheath, except through the capacitance to the outside shield. The voltage across the core insulation will then be so small that core insulation failures will not occur, unless the voltage between the outside shield and the sheath is large enough to puncture the coating. Thus, if the resistance of the outside shield were 1.1 ohms per mile (10-mil copper shield around 1.2″ cable) and if the impulse breakdown voltage of the coating were 20,000 volts, breakdown of the insulation would not be expected except for currents in excess of 130 kiloamperes when the earth resistivity is as high as 4000 meter-ohms. For a cable of 2″ diameter with a 10-mil copper shield, breakdown of the thermo-

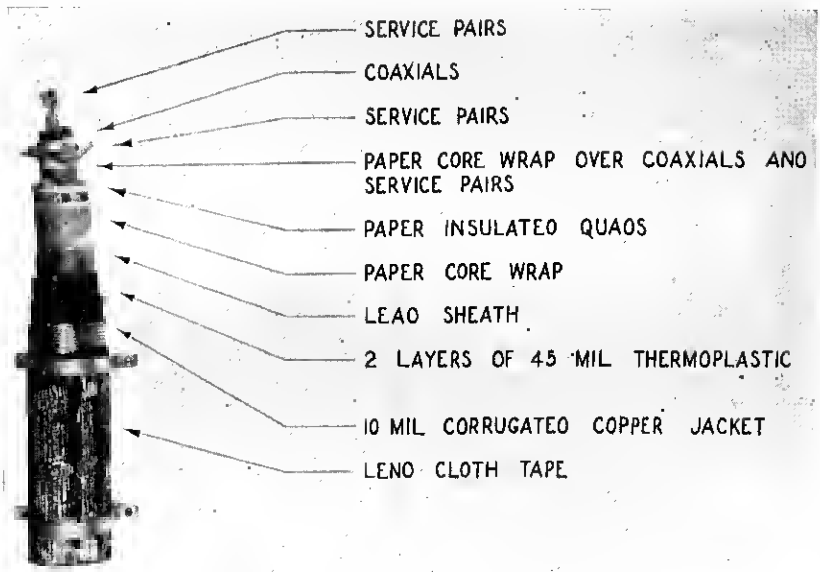

Figure 14—Thermoplastic covered, copper jacketed cable.

plastic insulation would not be expected except for currents in excess of 200 kiloamperes when the breakdown voltage of the coating is 20 kv and the earth resistivity is 4000 meter-ohms, or when the breakdown voltage is 10 kv and the earth resistivity 1000 meter-ohms. The above type of cable is also advantageous in that low-frequency induced voltages and noise due to static are reduced.

A cable of the above type is now being installed for a distance of about 180 miles along the Atlanta-Meridian route, where the effective earth resistivity varies between 1000 and 4000 meter-ohms and lightning storms occur frequently. The diameter of the lead sheath of this cable is about 2″, and the sheath is covered with two layers of thermoplastic each 45 mils

thick, with diametrically opposite seams having an overlap of about $\frac{1}{2}''$. The outside 10-mil copper shield is corrugated to facilitate bending and has a $\frac{1}{4}''$ overlap at the seam. The thermoplastic coating is flooded with thermoplastic cement to reduce moisture absorption. A photograph of this cable is shown in Fig. 14.

SUMMARY

Current in the sheath of buried cables, due to direct lightning strokes, strokes to ground near the cables or discharges between clouds, gives rise to voltages between the cable conductors and the sheath. The voltages are practically proportional to the square root of the earth resistivity and to the direct-current sheath resistance. For the latter reason they are substantially larger for carrier cables of the size now used than for the much larger voice-frequency cables. While direct strokes are usually most important, strokes to ground must be considered when the cables are of small size, even when the surface resistivity is low, provided the resistivity at greater depths is high. Under the latter conditions it is possible that for small cables, discharges between clouds over the cable may also cause failure.

For cables with thermoplastic or rubber coating the voltages between the sheath and the core conductors are much the same as for jute-covered cables. The coating of such cable is likely to be damaged by direct strokes and strokes to ground near the cable, in which case corrosion of the sheath may occur at such points.

Based on theoretical lightning expectancy curves, the incidence of lightning troubles increases faster than the sheath resistance or the earth resistivity. When the breakdown voltage of the core insulation is doubled by use of extra core wrap, or when shield wires are installed in situations where lightning damage is anticipated or has been experienced, a substantial reduction in lightning failures is to be expected. Shield wires will, however, not prevent damage to the sheath and the sheath coating.

Where the earth resistivity is very high and lightning storms occur frequently, doubled core insulation together with shield wires may not provide sufficient protection, even for cable of substantial size. Protection against various forms of lightning damage may then be secured by use of thermoplastic sheath coating of adequate dielectric strength together with an outside concentric copper shield.

References

1. E. D. Sunde: "Lightning Protection of Buried Cable," *Bell Telephone Laboratories Record*, Vol. 21, No. 9, May 1943.
2. B. F. J. Schonland: "The Lightning Discharge," Clarendon Press, Oxford, England, 1938 or "Thunderstorms and Their Electrical Effects," *Proc. of the Phys. Soc.*, Vol. 55, Part 6, No. 312, Nov. 1, 1943.

3. K. B. McEachron: "Lightning to the Empire State Building," *Electrical Engineering*, Dec. 1938 and Sept. 1941.
4. W. W. Lewis and C. M. Foust: "Lightning Investigations on Transmission Lines" VIII—*A.I.E.E. Trans.*, Vol. 64, 1945, p. 107.
5. H. Grünewald: C.I.G.R.E. 1939, Report 323.
6. C. F. Wagner and G. D. McCann: "Lightning Phenomena," *Elec. Engg.*, Aug., Sept. and Oct. 1941.
7. C. E. R. Bruce and R. Golde: "The Mechanism of the Lightning Stroke and its Effect on Transmission Lines," *Jour. I.E.E.*, Part II, Dec. 1941.
8. E. D. Sunde: "Surge Characteristics of a Buried Bare Wire," *A.I.E.E. Transactions*, Vol. 59, 1940 or *Bell Telephone System Monograph* B-1279.
9. S. A. Schelkunoff: "Electromagnetic Theory of Coaxial Transmission Lines and Cylindrical Shield," *Bell System Technical Journal*, October, 1934. Equation (5) is equivalent to Equation (82) of the above paper.
10. G. A. Campbell and R. M. Foster: "Fourier Integrals for Practical Applications," *Bell Telephone System Monograph* B-584, pair 807.
11. E. D. Sunde: "Currents and Potentials Along Leaky Ground Return Conductors," *Elec. Engg.*, December 1936 or *B.T.S. Monograph* B-970.
12. J. R. Carson: "Wave Propagation in Overhead Wires with Ground Return," *Bell Sys. Tech. Jour.*, October 1926. Z obtained by use of (34) and (35) and M by use of (36) and (37) of the paper referred to.
13. J. Riordan and E. D. Sunde: "Mutual Impedance of Grounded Wires for Stratified Two-Layer Earth," *Bell System Technical Journal*, April 1933 or *B.T.S. Monograph* B-726.
14. E. Hansson and S. K. Waldorf: "An Eight-Year Investigation of Lightning Currents and Preventive Lightning Protection on a Transmission System" presented at A.I.E.E. 1944 Winter Convention.
15. W. H. Alexander: "The Distribution of Thunderstorms in the United States, 1904–33," *Monthly Weather Review*, Vol. 63, 1935, page 157.
16. A. Matthias: "Modellversuche über Blitzeinschläge," *E.T.Z.* Aug. 12 and Sept. 9, 1937.
17. A. Matthias and W. Burkhardtsmaier: "Der Schutzraum von Blitzfang-Vorrichtungen und seine Ermittlung durch Modellversuche," *E.T.Z.*, June 8 and June 15, 1939.
18. C. F. Wagner, G. D. McCann and G. L. MacLane, Jr.: "Shielding of Transmission Lines", *A.I.E.E. Trans.*, Vol. 60, 1941, p. 313–328.
19. C. F. Wagner, G. D. McCann and C. M. Lear: "Shielding of Substations," *A.I.E.E. Trans.*, Vol. 61, 1942, p. 196–200.
20. P. L. Bellaschi, R. E. Armington and A. E. Snowden: "Impulse and 60-cycle Characteristics of Driven Grounds—II," *A.I.E.E. Trans.*, Vol. 61, 1942, p. 349.
21. R. Davis and T. E. M. Johnston: "Surge Characteristics of Tower Footing Impedance," *J.I.E.E.* Vol. 88, Part II, No. 5, Oct. 1941.
22. I. R. Eaton: "Impulse Characteristics of Electrical Connections to the Earth," *General Electric Review*, Oct. 1944, p. 41–50.
23. G. D. McCann: "The Effect of Corona on Coupling Factors between Ground Wires and Phase Conductors," *A.I.E.E. Trans.*, Vol. 62, 1943, p. 818–826.
24. G. Wascheck: "Earth Resistivity Measurements," *Bell Telephone Laboratories Record*, Vol. 19, No. 6, Feb. 1941.